Origin of Gibrat law in Internet: Asymmetric distribution of the correlation

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Although Gibrat's law and its generalized versions have been widely used, the organizing principle behind its phenomenological theory has been poorly studied for network-structured systems. More important, its fluctuation behavior, which contradicts the prediction of the preferential attachment (PA), indicates a nontrivial mechanism that goes beyond our present knowledge based on the traditional mean-field approach. Here, we take advantage of the rich data of the Internet and aim to identify the origin of Gibrat's law by studying the empirical fluctuation behavior. We show how the correlation between the fluctuations of the node degree increment affects the dynamics of the network. Specifically, if the distribution of the correlation is symmetric, the network evolves as the classical PA, while if such symmetry breaks, the fluctuation becomes macroscopically positively correlated and contributes to the emergence of Gibrat's law. These results indicate a local collective increase in the actual network evolution, which provides a new paradigm and understanding of the related microcosmic dynamics.

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I. INTRODUCTION

Our ability to predict and control a complex system relies largely on our knowledge of its organizing principle, and efforts have been devoted to constructing applicable models for decades [1–9]. The Gibrat's law model, as the most famous one, has received great interest and has been studied in all kinds of fields, such as population growth, economic fluctuation, firm evolution, and human dynamics [10–12]. Its wide application indicates the possibility of the existence of a universal law, which is of sufficient significance to require constant and exhaustive study.

Gibrat's law can be formalized by the following random multiplicative process:

$$k_{t+\Delta t} = R_t k_t, \tag{1}$$

where k_t can represent any quantity such as population, firm size, or node degree as in the present study. R_t is a random process, whose logarithmic value $r_t = \ln R_t$ is called the growth rate and is closely related to the microcosmic dynamics. In the classical version of the model r_t is assumed to be independent of the initial value k_t, so that its standard deviation conditional on k_t is a constant [13]. In recent years, the evidence accumulated in socioeconomic systems shows apparent inconsistency with this classical assumption. It has inspired a large number of studies on model modification, which led to a substantial development in socioeconomics using Gibrat's law [14–17].

In contrast, the application of Gibrat's law to the evolution of network structure has received far less attention. The related work can be traced back to the study on the World Wide Web by Huberman and Adamic [18] and the study on the fluctuation of the Internet by Goh *et al.* [19]. However, both of their studies are phenomenological, with no detailed description of how links are created and connected, which is far from a complete theory compared with other network models based on the PA rule [20]. Stanely *et al.* have proposed a variety of generalized Gibrat's law models based on the assumptions that the system is composed of some subsystems and each subsystem evolves as the classical Gibrat's law [15–17]. But their models seem inapplicable to network-structured systems because the knowledge of the microcosmic dynamics depends on understanding the origin of Gibrat's law at the node level in the first place rather than taking it as part of the assumption. Recently, Rybski *et al.* proved that the pure PA rule causes the conditional deviation of the degree growth rate decays with the initial degree as a power law of exponent -0.5 [12], which contradicts with the classical Gibrat law. They point out that this property is caused by the independency of the formation of links and indicate that the existence of the correlation can reduce the exponent, but in the end, they do not specify the identification and other details of this correlation.

While both Gibrat's law and PA cause similar proportionate effects at the mean-field level [6], the inconsistency of their fluctuation aspect indicates a nontrivial mechanism underlying the dynamics of Gibrat's law, which motivates us to ask two questions: (1) What mechanism is responsible for the emergence of Gibrat's law in a network? (2) How does it work on earth? To settle the two questions we study the fluctuation of the Internet, which once was identified to follow Gibrat's law. We find that the fluctuation is not universal but experiences a crossover transition from the PA phase to the Gibrat phase with the increase of the observed time scale. By taking advantage of this transition we implement a comparative analysis and identify that the emergence of Gibrat's law originates from the asymmetric distribution of the correlation between the fluctuation of the node degree increment. This paper is organized as follows. In Sec. II, we provide empirical evidence for the crossover transition in the fluctuation of the Internet. In Sec. III, we analyze the correlation of the fluctuation of the degree increment. We propose a possible theory of the emergence of Gibrat's law and validate it empirically with the discovery of the symmetry breaking of the correlation. In Sec. IV, we draw the conclusions.

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II. CROSSOVER TRANSITION IN THE FLUCTUATION OF THE INTERNET

Our Internet data come from the Oregon Route Views project [21]. They include snapshots of three different time scales, i.e., daily (30 days: 1–30 September 2006), monthly (36 months: January 2005 to December 2007), and yearly (15 years: 1998–2012). The original data are in the form of Border Gateway Protocol routing tables, from which an Internet graph can be constructed. As usual, each node represents a specific autonomous system (AS), while each edge is the logical link between the interconnected AS's, so that we get a network of size $\sim O(10^5)$ and of an almost constant average degree of about 4.5. The topological properties that we measured are stationary for all three time scales and are consistent with previous empirical studies [8]. Both the degree distribution and the average nearest neighbor degrees are power laws, written as $p(k) \sim k^{-\alpha}$ and $k_{nn}(k) \sim bk^{-\beta}$, with the exponents and the coefficient measured as $\alpha \approx 2.1$, $\beta \approx 4.5$, and $b \approx 630$, respectively. We also check the dynamics of PA as done in Ref. [22]. We find that for all three time scales, the linear PA $\Delta k \sim ck$ is always valid, with c measured to be about 0.1 for yearly data.

Our analysis of the fluctuation behavior focuses on the growth rate,

$$r_i(t) = \ln \frac{k_i(t+\Delta t)}{k_i(t)}, \tag{2}$$

where $k_i(t)$ is the degree of node i at time t, Δt is the observed time window, and $r_i(t)$ is the corresponding growth rate. The fluctuation property can be described by the standard deviation of r_i conditional on the initial degree $k_i(t)$, which is defined as

$$\sigma(k_i) \equiv \sqrt{\langle r_i^2 \rangle - \langle r_i \rangle^2}, \tag{3}$$

where $\langle \cdot \rangle$ represents the average over the same k_i. For the classical Gibrat's law Eq. (3) is a constant for any initial k_i, while for PA it decays as a power law with an exponent of -0.5. For the actual Internet, we study Eq. (3) for three different time scales, i.e., Δt represents 1 day, 1 month, and 1 year. As shown in Fig. 1(a), the fluctuation property changes with the increase of the time scales. For daily fluctuation, $\sigma(k)$ decays as a power law with an exponent of -0.5, just like the prediction of PA [12]. For monthly fluctuation, the small-degree region becomes flat, while the large-degree region remains unchanged. With the increase of Δt, the flat area extends gradually, and finally, Gibrat's law dominates the region of $k < 300$ for the yearly fluctuation. This result indicates that neither PA nor Gibrat's law characterizes the overall fluctuation of the Internet. They validate only a specific time scale. In addition, neither of them predicts any such transition at all. Note that our finding is different from those from human dynamics and firm growth, where a universal scaling law is reported [10,12,14]. It is also noteworthy that the discrepancy of the fluctuation for different Δt does not affect their PA effect. Actually, even if we reshuffle the process of the link formation, the linear PA still holds. However, when we apply the reshuffling treatment [12] for the yearly data, as shown in Fig. 1(b), Gibrat's law vanishes, and the fluctuation behaves as PA. This result indicates that for long-term evolution of the Internet, the independent formation of links assumed by the traditional PA rule is not valid. Indeed, the reshuffling process does not destroy the proportionate effect of PA and only removes any possible correlation.

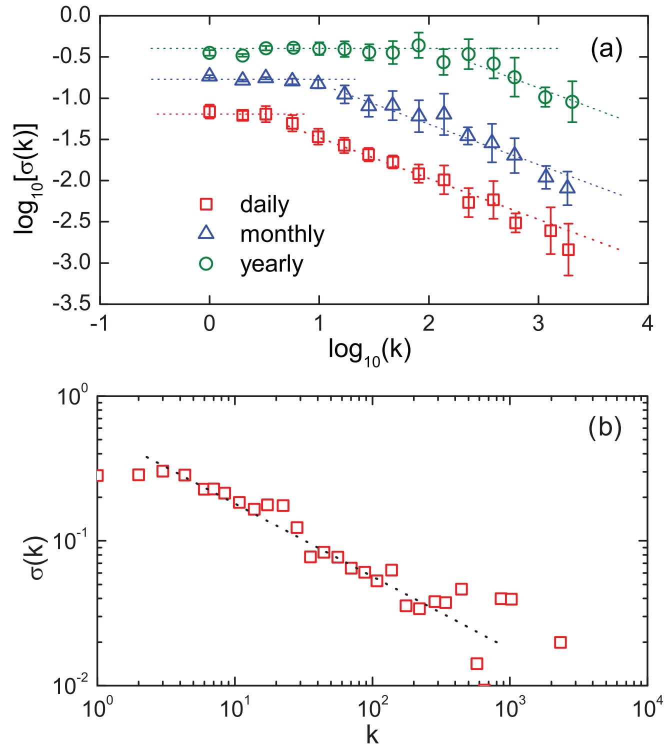

FIG. 1. (Color online) (a) The correlation between the conditional standard deviation $\sigma(k)$ and the initial degree k for three time intervals. For each time interval the dotted line in the flat part and in the decaying part represents a certain constant value and a certain power-law function with an exponent of -0.5. The fluctuation of the degree of growth rate experiences a crossover transition from the PA to Gibrat's law with the increase of the observed time interval. This transition starts in the small-degree region and gradually extends outward. (b) The correlation between the conditional standard deviation $\sigma(k)$ and the initial degree k for the reshuffled yearly data. The reshuffling process is similar to that in Ref. [12]. It is implemented by randomly exchanging two new added links during the 15-year period. It destroys any possible correlation between the formation of links while still maintaining the PA rule. In contrast to the yearly data in (a), the fluctuation decreases with a power-law exponent of -0.5. This result indicates the invalidation of the independent assumption of the PA rule.

III. SYMMETRY BREAKING OF THE CORRELATION BETWEEN THE FLUCTUATION OF THE DEGREE INCREMENT

Thanks to the rich data of all three time scales, we can make a comparative analysis to find any dynamical property related to their fluctuation. In the following analysis, we only use monthly and yearly data because daily data contain a larger fraction of data with unchanged degree, which might cause a too large statistical bias. Our discussion starts from the traditional PA rule and further leads to a relation between the fluctuation of the degree increment of a node and that of its neighbors. Then we propose a possible explanation for the emergence of Gibrat's law and try to validate it with the real data.

The traditional PA causes the proportionate effect that the increment of the node i degree follows,

$$\Delta k_i = ck_i, \tag{4}$$

where c is a universal constant for all nodes. Consider the sum of the degree increment of the neighbors of node i; we have $\sum_{j\in V(i)} \Delta k_j = c\sum_{j\in V(i)} k_j$, where $V(i)$ represents the set of the neighbors of node i. Since the average degree of the nearest neighbor in the Internet decays as $k_{nn}(k) \sim bk^{-\beta}$, we have

$$\sum_{j\in V(i)} \Delta k_j = cbk_i^{1-\beta}. \tag{5}$$

Along with Eq. (4), we have

$$\Delta k_i = C\left(\sum_{j\in V(i)} \Delta k_j\right)^{\frac{1}{1-\beta}}, \tag{6}$$

where $C = (bc^\beta)^{\frac{1}{1-\beta}}$. In the actual Internet, Eq. (6) is only valid in the mean-field situation. The real increment fluctuates around Eq. (6). Denoting δ_i as the fluctuation of Δk_i, the real increment reads as $\Delta k_i + \delta_i$. Now let us suppose not only the mean of the degree increment but also its actual random value satisfies Eq. (6); then we have

$$\Delta k_i + \delta_i = C\left(\sum_{j\in V(i)} \Delta k_j + \sum_{j\in V(i)} \delta_j\right)^{\frac{1}{1-\beta}}. \tag{7}$$

Considering the fluctuation is usually far smaller than its mean, we immediately come to the following relation:

$$\delta_i = G\left(\sum_{j\in V(i)} \Delta k_j\right)\sum_{j\in V(i)} \delta_j, \tag{8}$$

where $G(x) = \frac{dCx^{\frac{1}{1-\beta}}}{dx} = \frac{C}{1-\beta}x^{\frac{\beta}{1-\beta}}$. Note that the measured $\beta \approx 0.45$ leads to $\frac{\beta}{1-\beta} \approx 0.8 < 1$. The sublinear form of $G(x)$ indicates that it increases very slowly for large x. Therefore for a large range of $\sum_{j\in V(i)} \Delta k_j$, $G(x)$ can be approximated to be a constant $g = G(\langle\sum_{j\in V(i)} \Delta k_j\rangle)$, leading to the final linear correlation

$$\delta_i \sim g\sum_{j\in V(i)} \delta_j. \tag{9}$$

In Fig. 2, we plot the empirical result of the correlation between δ_i and $\sum_{j\in V(i)} \delta_j$. It follows a linear relation as Eq. (9) predicted. The slope in Fig. 2 is rather small, about 1.3×10^{-2}. One might question whether such a small slope is a physical reality or merely chance. However, our study indicates that the linear correlation occurs for every successive year without exception. Furthermore, by substituting the measured parameters b, c, β and the average $\langle\sum_{j\in V(i)} \Delta k_j\rangle \sim O(10^2)$, the theoretical slope can be directly calculated. Indeed, we find the calculated slope is of order $O(10^{-2})$ to $O(10^{-3})$, which is quite comparable to the empirical results.

The validation of Eq. (9) is useful for us to identify the correlation that occurs in the evolution of the Internet and to establish a possible explanation for the origin of Gibrat's law. Consider node i with degree k_i at time t that gains an

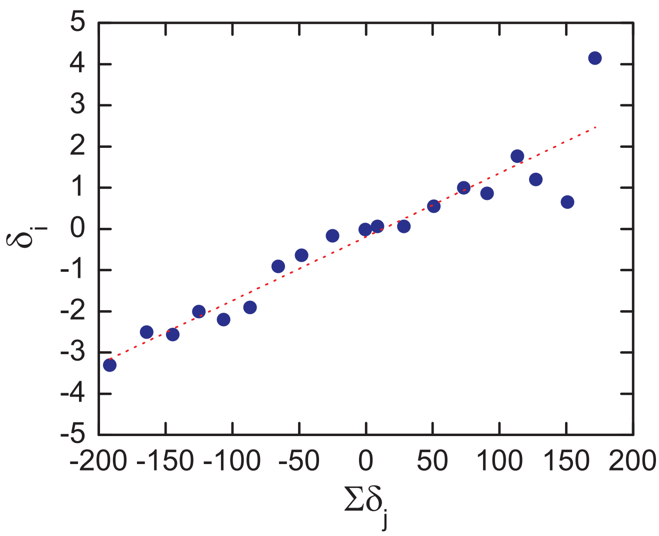

FIG. 2. (Color online) The correlation between the fluctuation of the degree increment of a node and that of its neighbors. It follows a linear relation $\delta_i \sim g\sum_{j\in V(i)} \delta_j$, with g measured to be about 1.3×10^{-2}, as fitted by the red dotted line. The theoretical value of g is calculated as $O(10^{-2})$ to $O(10^{-3})$, which is in agreement with the empirical measurement.

increment Δk_i after Δt. According to Eq. (2), the growth rate of its degree is $r_i(t) \sim (\langle\Delta k_i\rangle + \delta_i)/k_i$. By substituting $\langle\Delta k_i\rangle = ck_i$, the growth rate is

$$r_i(t) = c + \frac{\delta_i}{k_i}. \tag{10}$$

The conditional variance is written as $\sigma^2(k_i) \sim \frac{1}{k_i^2}\sigma^2(\delta_i)$. Using Eq. (9) and assuming the covariance is proportional to its correlation coefficient, we have

$$\sigma^2(k_i) \sim \frac{1}{k_i^2}\sum_{m,n\in V(i)} C(\delta_m,\delta_n), \tag{11}$$

where $C(\delta_m,\delta_n)$ is the correlation coefficient. If the creation of links is purely independent, there will not be a correlation between δ_m and δ_n, leading to $C(\delta_m,\delta_n) = 0$ for $m \neq n$. In this case only k_i terms of $C(\delta_m,\delta_n)|_{m=n} = 1$ exist, which causes $\sigma(k_i) \sim 1/\sqrt{k_i}$, i.e., the result of the PA rule. On the other hand, if each pair of δ_m and δ_n are correlated, which causes, for example, $C(\delta_m,\delta_n) \sim$ const, then the sum will be of order k_i^2, giving rise to the Gibrat property of $\sigma(k_i) \sim \text{const} \propto g$.

To confirm whether such a correlation exists and causes a similar effect, we calculate the sum of the correlation coefficient of the neighbors of node i whose degrees range from 20 to 200. (The details of the calculation are specified in the caption of Fig. 3.) In this range of degrees, the fluctuations in the monthly data and yearly data belong to the PA phase and the Gibrat phase, respectively. Therefore we can make a comparative analysis which can provide direct evidence for the connection between Gibrat's law and the proposed correlation. In Fig. 3, we plot the correlation between the degree k_i and $\sum_{m,n\in V(i)} C(\delta_m,\delta_n)$. As expected, from the theory for the Gibrat phase and the PA phase, for yearly data $\sum_{m,n\in V(i)} C(\delta_m,\delta_n)$ increases faster with k_i. Its power-law exponent is about 1.9, which is closer to 2, while for monthly

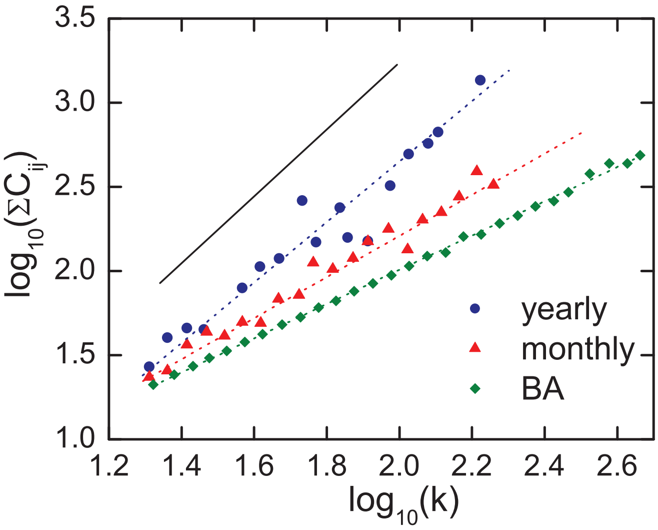

FIG. 3. (Color online) The correlation between $\sum_{m,n\in V(i)} C(\delta_m,\delta_n)$ and degree k_i, where $C(\delta_m,\delta_n)$ is a correlation coefficient of nodes m and n, which belongs to a neighbor of i. The calculation procedure of $C(\delta_m,\delta_n)$ is as follows. First, we calculate the fluctuation δ of the degree increment of each node for every successive years. Then we get a resemble $\{\delta\}$ of length 14 for each node. We regard this resemble as the sample produced by the random variable δ_i. We can calculate the correlation coefficient from pairs of these samples and finally get the present result. We find that for yearly data $\sum_{m,n\in V(i)} C(\delta_m,\delta_n)$ increases faster with k_i with a power-law exponent of about 1.9 (as fitted by the blue dotted line with circles), which is closer to 2, while for monthly data the exponent is about 1.2 (red dotted line with triangles), which is closer to 1 and is significantly smaller than that of yearly data. The exponent of the correlation for the BA model is 1 (green dotted line with diamonds), which is consistent with the purely independent formation of links assumed by the PA rule. The black solid line has a slope of 2. Note that the data points for yearly data are shifted a little for better visualization.

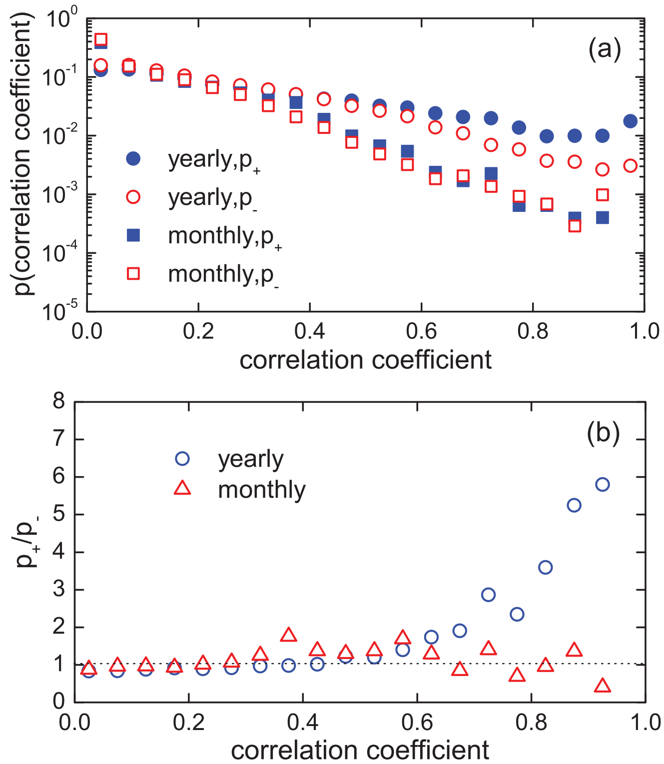

FIG. 4. (Color online) (a) The distribution of the correlation coefficient for yearly data and monthly data. p_+ and p_- represent the distribution of the positive and negative correlation coefficients, respectively. They are both plotted according to the absolute value of $C(\delta_m,\delta_n)$ to better illustrate their symmetric property. p_+ and p_- for monthly data collapse nicely, indicating the distribution is symmetric. In contrast, p_+ becomes apparently larger than p_- when $|C(\delta_m,\delta_n)| > 0.6$ for yearly data, evidencing the symmetry breaking of the correlation. (b) p_+/p_- vs the absolute correlation coefficient for monthly and yearly data. For the monthly period p_+/p_- stays around 1, while for the yearly period p_+/p_- starts to increase at 0.6 and gradually becomes significantly larger than 1.

data the exponent is about 1.2, which is closer to 1 and is significantly smaller than that of the yearly data. The correlation for a purely Barabási–Albert model (BA model) is also plotted. The exponent is 1, consistent with the assumption of independent formation of links.

The above empirical result provides evidence for the connection between Gibrat's law and the correlation between the fluctuations of the degree increment of neighboring nodes. This correlation reveals a very different picture from the traditional description. It indicates that the evolution of the Internet tends to occur collectively in a local area. In other words, nodes in a neighborhood tend to gain (lose) more or less links simultaneously, leading to a locally rapid increase.

Having determined the identification of the correlation, we turn to investigate how in effect it works. We first examine the distribution of these correlation coefficients $C(\delta_m,\delta_n)$ with $m,n \in V(i)$ for all nodes i. As shown in Fig. 4(a), the actual correlation coefficients of both monthly data and yearly data are distributed in a large area and display exponential-like decay. Further inspection indicates an asymmetric distribution for yearly data, i.e., the number (distribution) of the strong positive correlation coefficient (>0.6), denoted as p_+, is larger than that of the negative correlation, denoted as p_-. In contrast, for monthly data p_+ and p_- collapse, indicating a symmetric distribution. This property can be further demonstrated by studying the fraction of p_+ and p_-. For a symmetric distribution, the fraction is close to 1 for all absolute values of the correlation coefficient, as shown in Fig. 4(b) for monthly data. However, for yearly data, the fraction begins to increase at an absolute value of the correlation coefficient of 0.6 and becomes significantly larger than 1 for a stronger correlation. More significantly, this property is valid not only on the macroscopic level but also for different degrees k_i. For yearly data all p_+/p_- of the three different bins of degrees start to grow at an absolute correlation coefficient of 0.6, and they increase in the same manner. As indicated in Fig. 5(a), the data collapse nicely. Actually, not only p_+/p_- but also $p_+ - p_-$, p_+, and p_- collapse for different degrees. On the other hand, p_+/p_- of different degrees for monthly data stay around 1 [Fig. 5(b)], indicating the validation of the symmetric property for all nodes.

The similar symmetry-breaking property of the correlation coefficients for all nodes reveals the real contribution of $C(\delta_m,\delta_n)$ to the emergence of Gibrat's law. Consider a

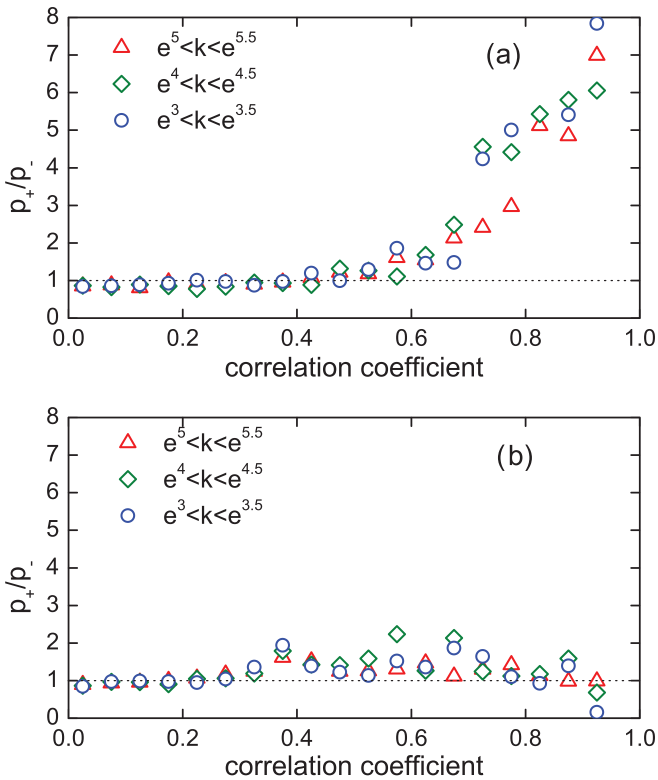

FIG. 5. (Color online) (a) p_+/p_- vs the absolute correlation coefficient for different degrees for yearly data. The asymmetric properties are almost the same for various degrees. (b) p_+/p_- vs the absolute correlation coefficient for different degrees for monthly data. All of them stay around 1, indicating a symmetric distribution of $C(\delta_m,\delta_n)$.

node i with degree k_i and let $m,n \in V(i)$; the number of $C(\delta_m,\delta_n)|_{m\neq n} = \varepsilon\ (-\varepsilon)$ is asymptotically $k_i^2 p_+(\varepsilon)\ [k_i^2 p_-(\varepsilon)]$, while the corresponding number of $C(\delta_m,\delta_n)|_{m=n} = 1$ is k_i. Therefore the sum of the correlation coefficient reads

$$\sum_{m,n\in V(i)} C(\delta_m,\delta_n) = \sum_{m\neq n} C(\delta_m,\delta_n) + \sum_{m=n} C(\delta_m,\delta_n)$$
$$\sim k_i^2 \int_0^1 \varepsilon[p_+(\varepsilon) - p_-(\varepsilon)]d\varepsilon + k_i. \quad (12)$$

For a symmetric distribution, $p_+ - p_- = 0$; thus $\sum_{m,n\in V(i)} C(\delta_m,\delta_n) = k_i$, leading to $\sigma(k) \sim 1/\sqrt{k_i}$. For an asymmetric distribution, since $p_+ - p_-$ is the same for all k_i, $\int_0^1 \varepsilon[p_+(\varepsilon) - p_-(\varepsilon)]d\varepsilon$ equals a nonzero constant. Then it finally comes to $\sigma(k) \sim \text{const} \propto g$. Therefore we conclude that it is the symmetry breaking of the correlation between the fluctuations of the degree increment that causes the emergence of Gibrat's law. Note that the number of correlated neighbors to sustain Gibrat's law increase as $O(k^2)$. Thus larger degree nodes need more time to accumulate to display the effect of Gibrat's law, which provides partial understanding of the period-related crossover transition.

IV. CONCLUSION

We have studied the fluctuation of the Internet by analyzing the conditional standard deviation of the degree growth rate. We find that the fluctuation exhibits a crossover transition from the PA phase to the Gibrat phase with the increase of the observed time scale. To uncover the origin of Gibrat's law, we study the correlation between the fluctuations of neighbors' degree increments. We find in the PA phase that the distribution of the correlation is symmetric, while in the Gibrat phase such symmetry breaks, leading to a macroscopic positive correlation and contributing to the emergence of Gibrat's law. These results indicate that the evolution of the Internet tends to occur collectively in a local area, which is not captured by the traditional mean-field description.

There have been numerous arguments on the validation of the PA rule, ranging from data collection to the analysis method [23]. While the classical PA describes in detail how links are formed at the individual (node) level, the resolution limitation of the data analysis allows empirical examination based on only the mean-field approach [22]. By tacitly approving the independence assumption of PA, these demonstrations seem reasonably to be generalized to the individual scale. However, our present finding of the unignorable correlation indicates that this evidence and reasoning can no longer be considered as a convincing justification. For systems where a similar phenomenon is observed [24], the microscopic mechanism might need further investigation.

ACKNOWLEDGMENT

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