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# Multi-message topic dissemination probabilistic model with memory attenuation based on Social–Messages Network

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Current researches give priority to the diffusion of single message, but the diffusion of multi-messages at the same time in the actual network also exists. The diverse correlation of the messages will influence each other in the diffusion. It should be taken into consideration. This paper works to make a definition to the framework of Social–Messages Network. Based on it, a multi-message topic dissemination probabilistic model with memory attenuation is put forward, which introduces the correlation among messages. We adopt a simple learning strategy to gain the diverse correlation of messages. Then, the numerical simulation is utilized to analyze the model, whilst the relationship of the model parameter with the scope of the topic diffusion and the spread speed are studied and analyzed. With the related discussion data on Twitter, an empirical study is made to the model and the diffusion progress of the message is anticipated, which suggested that the anticipation is fundamentally in line with the actual data, and the estimated value of our model is closer to the reality than the classic diffusion model. Study on the topic diffusion will be conducive to the understanding and the anticipation of the multi–messages spread.

*Keywords*: Social network; information dissemination; multiple messages; probabilistic model.

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## 1. Introduction

With the rapid development of Internet technology and the emergence of online social networks, the speed of information dissemination becomes faster, the influence extends wider, and the amount of information is becoming larger. Researchers make extensive use of complex networks to describe the dynamics of propagation in real-world, while the information transmission in the network is also a complex dynamic process, which is influenced by many factors, such as the topology of the network, the influence of the information publisher,[1] the interest preference of the user, the characteristics of the information itself and so on. Understanding the process of propagation in complex networks is a core issue in the field of network science.[2] Much of the studies in this respect are concerned with spreading processes such as the diffusion of innovations,[3,4] the propagation of rumors,[5,6] information diffusion in a society through the word-of-mouth[7] or the diffusion of influence[8,9] in the network. Therefore, the study of information dissemination in social networks is of great significance to network marketing, rumor control, Internet public opinion monitoring and so on. For example, Wang *et al.* discussed a logistic prediction model based on in online social network information dissemination[10–12]; Trpevski *et al.* explored the spread of rumors in multiple networks.[13]

The research of the transmission in the network originated from the spread of infectious diseases. They have successively proposed infectious disease models such as SIR, SIS and SEIR,[14,15] and subsequently explored the spread of infectious disease models in small-world and scale-free networks.[16,17] But compared with the spread of disease, information dissemination[18,19] is more intricate because it will be influenced by psychological factors such as individual social strengthening and homogeneity.[20–22] Furthermore in the actual situation of information dissemination, memory plays an important role in it. Considering the memory effect, some complex non-Markov models are proposed[22–26]; for example, Watts *et al.* proposed an SIR-extended propagation model that introduces memory effects in the process of information dissemination and accumulates information received from neighbors[26]; Shu further explored the effect of memory on information dissemination.[22] The effect of memory was introduced into these models, but most of them ignored the attenuation of memory. However, in the process of spreading information in real social networks, the communication or dissemination of the past will be forgotten or become less important over time.[27]

The models mentioned above mostly consider the transmission behavior of single message or single infectious disease in the network, but there is a phenomenon that a large number of messages co-propagate in the real network, for example, rumors, the spread of multiple diseases in the crowd; a variety of computer viruses spread on the computer network. Multi-messages propagation is more complex than single message propagation because there may be multiple interactions among messages. There may be a competitive or mutually reinforcing relationship among messages. In the study of multi-messages transmission, most of the research is based on the classical infectious disease model, which is focused on the interaction between the two viruses. For example, Newman discussed the critical value problem of two co-propagating pathogens[28]; Ahn *et al.* proposed a new virus propagation model according to the SIS model for two viruses (virus 1 will induce virus 2, but virus 2 will in turn inhibit virus 1).[29] But in the context of real social networks, when a topic is being discussed, users with many different perspectives are involved. As a result, hundreds of new messages can be generated at the same point in time, and the correlation among them is very different. Messages with the same views may promote each other. However, in previous studies, the phenomenon of multi-messages propagation with the same topic has been largely ignored. Therefore, systematic research is needed to understand the influence of multi-messages simultaneous diffusion on social propagation dynamics.

Based on this, this paper researches the propagation of multi-messages under the same topic. In order to further study the role of attribute features in the social network such as network link prediction, Gong *et al.* introduced the attribute features into the social network framework and proposed a new network framework Social–Attribute Network.[30,31] Based on this idea and social network, this paper introduces the relevance among messages and proposes a new network framework Social–Messages Network (SMN), which includes social sub-network and message sub-network. The framework is mainly aimed at multi-messages propagation, which can take into account the different correlation between two messages. The spread of one or two messages in the network can be extended to study the common propagation of multi-messages. Using this framework will contribute to researchers explore the propagation of a large number of messages at the same time in real social network and the outbreak mechanism of topics. And in this framework, we can study the spread and outbreak mechanism of the topic, and study how many messages are transmitted together in social network. Based on this network framework, this paper proposes a multi-message topic dissemination probabilistic model with memory attenuation, and introduces the effect of interaction among multi-messages under the same topic. The concatenation between messages represents the influence between the two messages, and the weights of the edges represent the intensity of the interaction between the two messages. Using simulation analysis and empirical evaluation, we verified that the model can predict the multi-messages propagation under the topic. Compared with the classical propagation model, the prediction of our model is more suitable for the actual situation.

In this paper, we extend social network to SMN, and investigate the spread of topic in the expanded SMN, which is helpful to understand and predict the multi-messages propagation. Section 2 defines the SMN framework and the model formation, in Sec. 3, we carefully analyze the experimental results and we conclude this paper in Sec. 4.

## 2. Definition of the Model

### 2.1. *Social–Messages Network*

In this section, we give a detailed introduction to the SMN and the significance of the parameters therein.

The SMN is an argument social network framework that includes a social sub-network and a message sub-network. Figure 1 shows an example SMN. First, a directed unweighted social network $G$ is constructed. The nodes in this network represent the users (called social nodes) and represented by $V_s$; The existence of directed links between social nodes represent the relationship between users (called social links), which are defined as $E_s$. Then, a message sub-network is established by users publishing messages in the social sub-network $G$. Nodes in message sub-network represent messages (called message nodes), which are defined as $V_m$. Edges between messages nodes are called message links, defined as $E_m$. Message links are undirected weighted edges. The weights of message links indicate that the correlation between the messages is strong or weak (If there is no continuous edge between the two messages, the correlation between them is 0 and they are not related.) Finally, when the node $u$ in social sub-network publishes or reposts the message $m$, an undirected edge is created between $u$ and $m$ nodes in the SMN. This edge is defined as the forwarding links and denoted as the $E_r$. Therefore, the SMN can be defined as SMN $= (V_s, V_m, E_s, E_r, E_m)$. The SMN includes a message sub-network and a social sub-network, and the two sub-networks are connected by forwarding links. Note that, in this framework, the message sub-network cannot exist independently of the social sub-network. Only when the user in the social sub-network published or reposted the message, the corresponding node and links appear in the message sub-network.

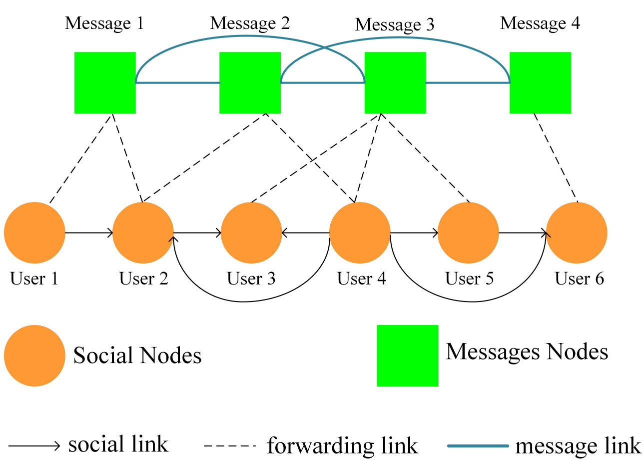


Fig. 1. (Color online) SMN.

The network framework can be used to research information dissemination and extend the propagation model in social network to SMN. It also helps in the study of multi-messages propagation in the network. It can better study the propagation and outbreak mechanism of topics, and can be used to study the impact of the relevance of messages on the spread of the network.

### 2.2. *Propagation mechanism*

At each time step, each individual adopts one of four states for each message: (i) $S$ state — the individual has not yet read the message, analogous to the susceptible state of the SIR model. (ii) $A$ state — the individual is aware of the message but not willing to repost it. (iii) $I$ state — the individual is interested in the message and then transmits it to his/her neighbors, analogous to the infective state of the SIR model. (iv) $R$ state — after transmitting the message, the individual will lose interest and never diffuse it again, analogous to the recovered state of the SIR model. The dynamic behavior of forwarding or publishing a message in this paper is regarded as a spreading behavior of the message.

Note that in a real social network, after a user propagating a message, it takes a certain amount of time to be read by his/her neighbors. It is not immediately possible to read it. Therefore, let $\lambda$ denote the reading probability that a user will read a message propagated by his/her neighbor.

For the user to propagate each message, we define the propagation mechanism as follows:

(i) If an $I$-state node is in contact with an $S$-state node, the $S$-state node will become the $A$-state node with reading probability $\lambda$.
(ii) The $A$-state node will become the $I$-state node with the transmitting probability $p_{k,m}(t)$, and the transmitting probability $p_{k,m}(t)$ is described in detail in Sec. 2.3.
(iii) The $I$-state node will not propagate endlessly, and will stop propagating within $R\_time$ time steps after transmission, then it will become an $R$-state node. Here, we define $R\_time$ as the moment of recovery.

### 2.3. *Multi-message topic dissemination probability model with memory attenuation*

For user $k$, we define $C_k$ as his/her following list set. The higher the proportion of users who propagate message $m$ in $C_k$, the higher the transmitting probability that user $k$ will spread the message. In addition, the user's memory is attenuated,[22] for instance, at time $t$, user $k$ receives message $m$, so this message $m$ has the greatest impact on user $k$, that is, the user $k$ has a higher transmitting probability to propagate the message $m$. If the user does not transmit message $m$ immediately, the user's memory of message $m$ gradually decreases at the next moment. In other words, the impact of the message on the users gradually weaken to zero with time passed. This paper defines the memory time window — $H$ as the user's memory time for each message being read (we assume that the memory time window $H = 24\,\mathrm{h}$, one day). The message which the time of reading beyond the time window is no longer considered to affect the user, and the cumulative effect of each message being forwarded to the user within the memory time window is calculated.

At time $t$, the probability that user $k$ retweets message $m$ is $p_{k,m,\mathrm{bottom}}(t)$ given as

$$p_{k,m,\mathrm{bottom}}(t) = \sum_{\tau=t-H-1}^{t-1} W(t-\tau) f_m(\tau), \tag{1}$$

$$f_m(\tau) = \frac{n_m(\tau) - n_m(\tau-1)}{N_{C_k}}, \tag{2}$$

where $\tau$ represents the $\tau$ moment and $t$ represents the current moment. $n_m(\tau)$ is the number of users who propagate message $m$ in $C_k$ at time $\tau$ and $N_{C_k}$ is the total number of users of $C_k$ and $W$ is the memory function given by

$$W(t-\tau) = \frac{\varepsilon}{t-\tau}, \tag{3}$$

where $\varepsilon$ is the factor of memory. When the reading interval $(t-\tau)$ is larger, the memory function $W$ becomes smaller.

In the message sub-network of the SMN, the edges of the message nodes reflect the relevance of the messages. To simplify the model, this paper uses a simple learning strategy to obtain the correlation among messages, i.e. in the process of propagation, at each time step, the model learns the results of the past time step to obtain a new message sub-network. By learning the new sub-network, we can learn the relevance of the message from the process of transmission and extrapolate the concatenation between messages and the weights of the edges. We define when a user has reposted two different messages (message $i$ and message $j$), these two messages are joined together in the sub-network.

Then, considering the correlation among messages, we define $\varphi_{k,m}$ to represent the weights of messages sub-network as follows:

$$\varphi_{k,m} = \frac{\sum_{j \in \text{the messages that reposted by user } k} \sigma_{mj}}{N_{k,t}}, \tag{4}$$

which indicates the influence of other messages when user $k$ propagates messages $m$ at time $t$, where $\sigma_{m,j}$ is the similarity between message $m$ and message $j$, and $N_{k,t}$ indicates the number of messages that user $k$ reposts at time $t$.

This paper uses Salton Cosine to calculate the similarity between two messages. It is defined as follows:

$$\sigma_{mj} = \frac{n_{mj}}{\sqrt{k_m k_j}}, \tag{5}$$

where $n_{mj}$ is the number of users who have simultaneously reposted message $m$ and message $j$ at time $t$. $k_m$ is the total forwarding amount of the message $m$ at time $t$. $k_j$ is the total forwarding amount of the message $j$ at time $t$.

Using the correlation among messages, $p_{k,m,\text{bottom}}(t)$ is modified to obtain a new transmitting probability $p_{k,m}(t)$. We map $\varphi_{k,m}$ to a Gaussian function, let $\varphi_{k,m} \in (0,1)$, and the Gaussian function has an area approximately equal to 1 in $[-3, 3]$. Therefore, after the correction, the transmitting probability $p_{k,m}(t)$ for user $k$ to forward message $m$ is defined as follows:

$$p_{k,m}(t) = \begin{cases} 1 - (1 - p_{k,m,\text{bottom}}(t) \times \text{Gauss}(3\varphi_{k,m}) & \text{if } \varphi_{k,m} > 0, \\ p_{k,m,\text{bottom}}(t), & \text{if } \varphi_{k,m} = 0. \end{cases} \tag{6}$$

## 3. Experimental Results

### 3.1. *Scale-free network*

In this section, we present results on BA network and set the network size, newly added edges and mean degree are $N = 1000$, $m = 6$ and $\langle k \rangle = 5.989$, respectively. The initial number of messages in the network is $N_{\text{msg}} = 20$, social nodes propagate different messages at the same time. Note that the selection of 20 social nodes is done randomly. Then, we separately discuss the effects of different parameters. At first, the influence of the memory parameter $\varepsilon$ on the propagation process is analyzed on the time scale. As shown in Fig. 2, when $\lambda$ is fixed, different $\varepsilon$ has different effects on propagation. With the increase of the memory parameter $\varepsilon$, the final reposting number of the topic ($R(\infty)$) gradually increases, and the propagation speed also slightly rises. According to the function $W(t-\tau) = \frac{\varepsilon}{t-\tau}$, the strength of memory and the speed of memory attenuation each time step have positive correlation with the parameter $\varepsilon$. If parameter $\varepsilon$ is rather large, it indicates that the users are more interested about this topic and have stronger memory, and it means that they have more possibility to repost this topic and make this topic propagate faster and wider.

Figure 3 shows the relationship among $R(\infty)$ and $\lambda$, $\varepsilon$ in the steady state (i.e. for the topic, all users are in $R$ state or $S$ state). We can obtain the following conclusions: (1) In steady state, the total forwarding amount of topic increases when we increase

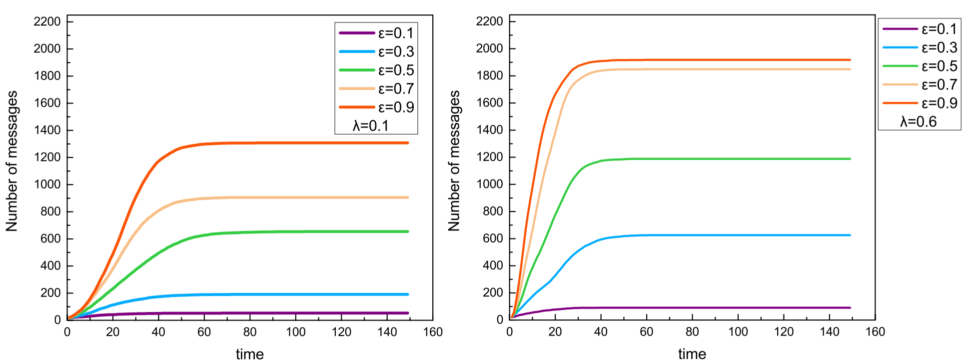


Fig. 2. (Color online) Different parameters $\varepsilon$ over time, the number of topics discussed.

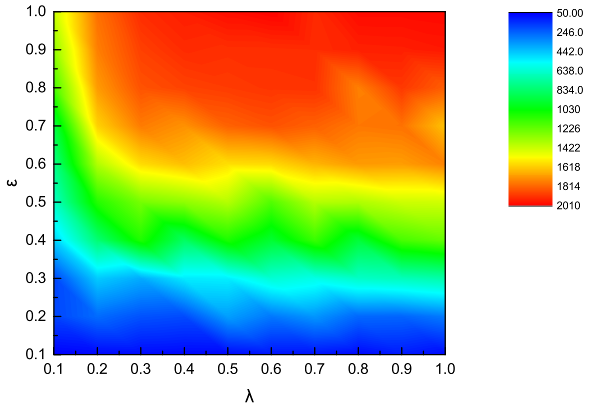


Fig. 3. (Color online) Steady state under different parameters, the number of topics transmitted.

the parameter $\varepsilon$. However, the total number does not reach the maximum theoretical value ($N \times N_{\mathrm{msg}}$). It is because the preference dependency strategy is used to construct a directed BA network, and the new edges are outlinked. Thus, the nodes at the beginning of the construction are generous nodes with large indegree (called initial nodes). The newly added node pays attention to the large node, but the large node does not follow these newly nodes. This is also more in line with the actual connection in online social network. Fans are concerned about stars, but stars rarely follow their fans. Messages sent from new nodes will not be transmitted to the initial nodes. In the other words, this message will not spread throughout the network. (2) The change in reading probability parameter $\lambda$ has little effect on the total forwarding amount of topic when $\varepsilon$ is rather small (i.e. $\varepsilon < 0.2$).

As shown in Fig. 4, we analyze the influence of the parameter $\varepsilon$ on the propagation speed, note that for each parameter, we count the 90% of the number of users in stable states divided by the time step required to reach 90% of the population of the final steady state as the propagation rate. Obviously, the larger the parameter $\varepsilon$ corresponding to the faster propagation rate. Because the larger the parameter $\varepsilon$ means that message being read has a greater impact on the user, that is, the user is more interested in this message, the transmitting probability for the user of this message will also increase. When the transmitting probability at each time step increases, the rate of messages transmission rises.

Then, we discuss the influence of different behavioral activity times on $R(\infty)$. Figure 5 shows the percentage of the total number of users participating in the topic, that is, the proportion of $R$-state users in the network, under different parameters when $R$_time $= 3$, 6 and 12, and the propagation reaches a stable state. (1) When the parameter $\varepsilon$ is large and the reading probability $\lambda$ is small (such as $\varepsilon > 0.6$, $\lambda < 0.3$), the change of $R$_time has an obvious influence on the proportion of users in $R$-state in steady state, and the propagation range will become wider with the increase of $R$_time. (2) According to Eq. (3), when the parameter $\varepsilon$ is too small (i.e. $\varepsilon < 0.2$ in

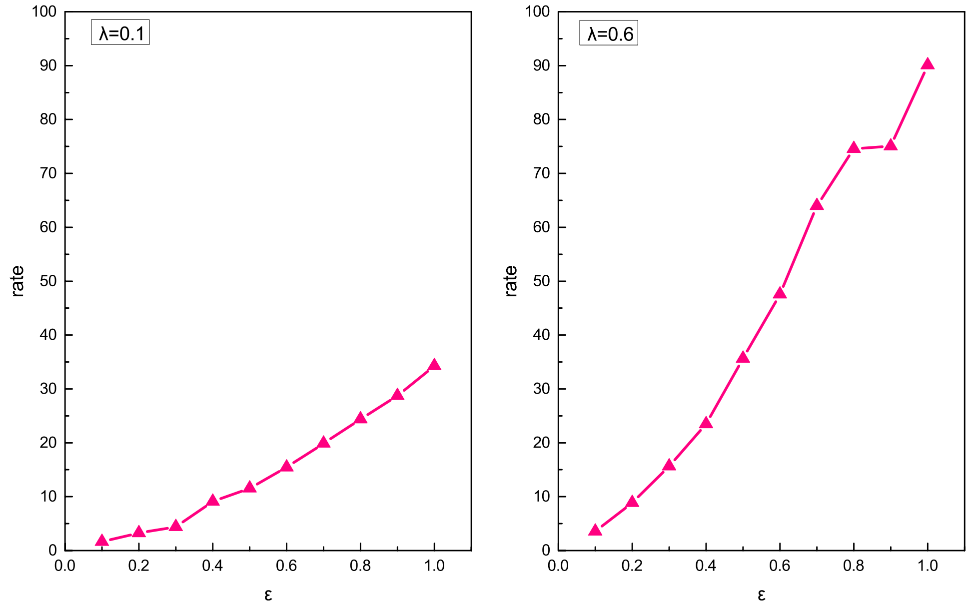


Fig. 4. (Color online) The relationship between the forwarding rate and the parameter $\varepsilon$.

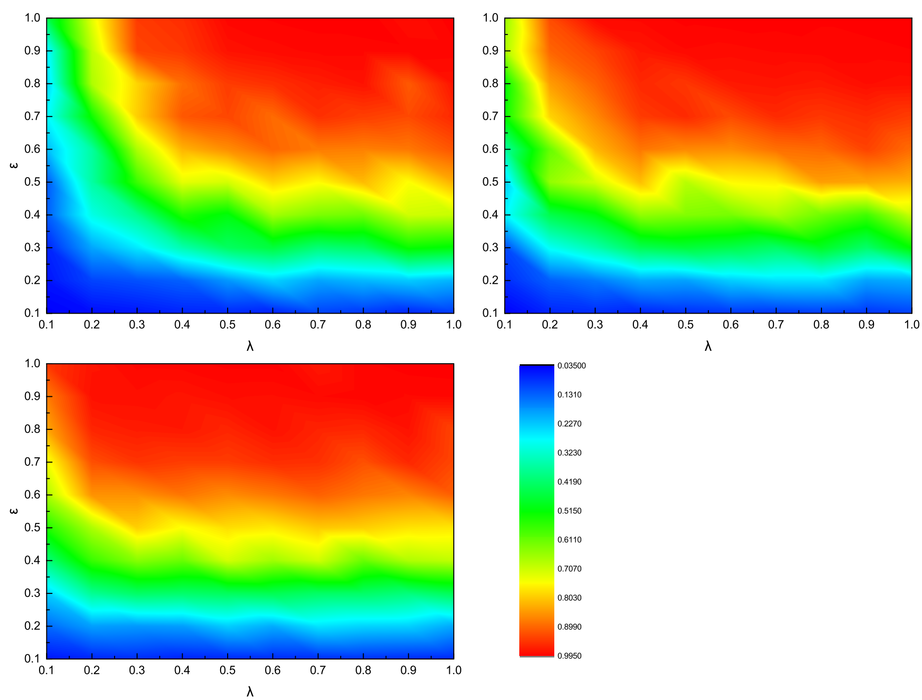


Fig. 5. (Color online) Active time of different activities, the number of participants in a stable situation, from left to right from top to bottom $R$_time, respectively, 3, 6, 12.

this paper), the message is not attractive to users and messages forwarded by friends have no obvious effect on users. Naturally, the transmitting probability is small. Even if increasing the $R_$time, the change of the propagation range is still not obvious. (3) When the reading probability $\lambda$ is large, i.e. $\lambda > 0.6$, the message retweeted by friend is more easily received by the user. During the active period of the behavior, the probability that the message has not been accepted is $(1-\lambda)^{R\_\mathrm{time}}$. When $\lambda = 0.6$, $R\_\mathrm{time} = 3$, the value of this probability is 0.064. In other words, the probability that the behavior of the user does not have any influence on his/her friend is very small. Therefore, before the action becomes inactive, the impact of this action had spread across the network, so the change of $R_$time has a little effect on the scope of the dissemination.

### 3.2. *Twitter network*

In this section, a Twitter network is used as the social sub-network. The number of user nodes is $N = 5000$ and the number of edges between users is 185 433. The number of messages at the initial time of the network is still $N_{\mathrm{msg}} = 20$.

Using the Twitter network, we consider the impact of the correlation among messages on the propagation in real networks. Figure 6 shows the number of discussions about the topic in the network over time, taking into account the correlation among messages or not. It can be clearly seen that in the initial stage of propagation, the number of messages on the topic is relatively close in both situations because the model in this paper obtains the correlation among messages by learning the propagation in the past time steps. The correlation value is small at the beginning of the propagation. However, with the evolution of diffusion, the correlation among messages has a significant effect on the promotion of propagation. We can find when we consider the correlation among messages, the topic will spread the entire network

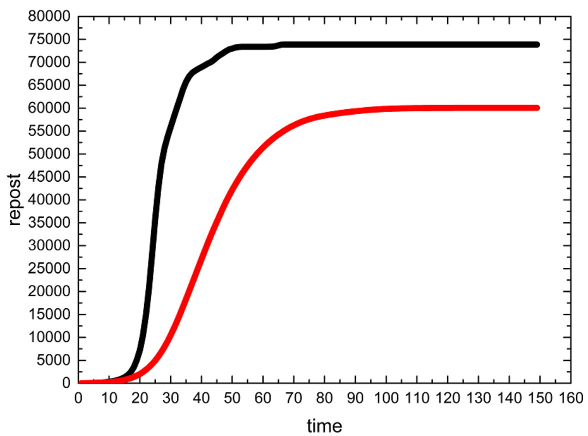


Fig. 6. (Color online) Consider the influence of propagation by the correlation of messages. Black line: Consider the correlation among messages. Red line: Regardless the correlation among messages.

faster, people pay more attention to the topic, and the number of messages in the final time steps is much higher than the case of not considering the correlation among messages.

### 3.3. *Compare with real social network*

The Twitter empirical data used in this paper relate to a match against Arsenal FC and Manchester City FC in the Premier League.[32] The whole topic lasted from December 14, 2015 to December 23, 2015, involving 4911 users. This paper analyzes and discusses the reposting situations of 210 different messages published by different users in this topic. This topic lasted for 220 h. Figure 7 shows the number of messages published per hour on this topic and over time, new messages are added to the message sub-network. In our model, we continuously update the SMN in the model based on the evolution of the message network in the actual situation.

The specific modeling and evolution processes are as follows:

(i) Construct the social sub-network according to the attention relationship among users.
(ii) Create an initial message sub-network based on the $N\_m(0)$ messages that exist at the time of the start of the propagation.
(iii) In each propagation time step $t$, update the number of messages $N\_m(t)$ at this time and constantly update the SMN.
(iv) Simulate user forwarding. At each time step $t$, for each message $m$ in the message sub-network, traverse the whole social sub-network and the message sub-network and calculate the transmitting probability of the message; if the user forwards different messages, they added the messages in the message sub-network to update the SMN.
(v) Repeat (iii)–(iv) steps ceaselessly until the entire message sub-network evolves.

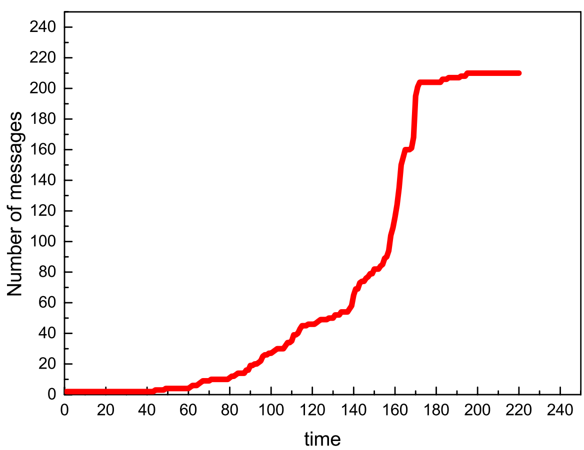


Fig. 7. (Color online) The number of messages published.

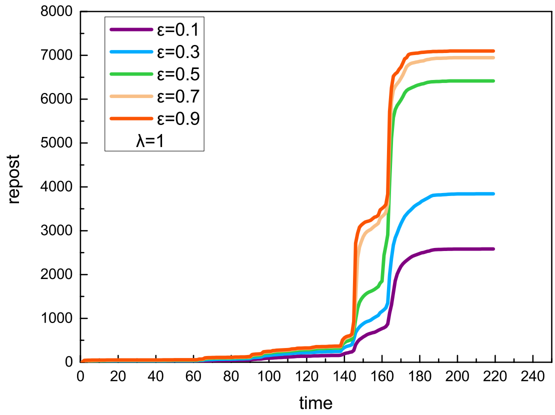


Fig. 8. (Color online) Different $\varepsilon$, the number of messages in the network.

Messages in an online social network typically have a very strong timeliness. The time interval between publishing and no longer receiving any forwarding is relatively short, and we define the interval as the life cycle of the message. Most messages in social networks have a short life cycle. In this section, we define the time window of the memory function and the time of recovery ($R$_time) as 24 h a day. Figure 8 shows the effects of different parameter $\varepsilon$ on the model. With the increase of $\varepsilon$, each user has a deeper impression on the messages which are forwarded or published by his/her friends. It is easier for users to forward messages that they are interested in or impressed with, so the final quantity of forwarding increases and the range of transmission expands.

Furthermore, in order to fit the Twitter empirical data, we change two key parameters in the model, the memory parameter $\varepsilon$ and the reading probability parameter $\lambda$, which represent the memory strength of user and the probability of users to read the related topics, respectively. As shown in Fig. 9, the predict condition of

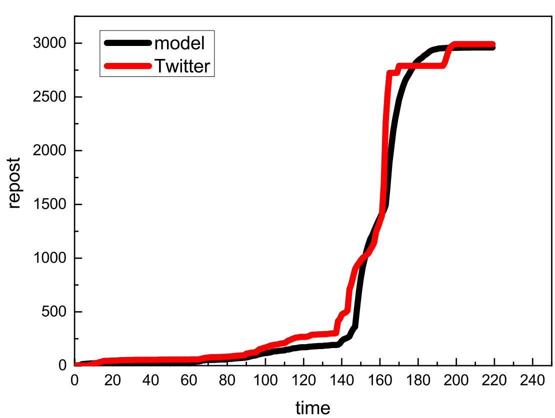


Fig. 9. (Color online) Comparison between the propagation by model and actual situation.

the model is agreed well with the Twitter empirical data in the parameter $\varepsilon = 0.16$, $\lambda = 1$. We should emphasis that the setting of parameters is corresponding to the real social networks. On the one hand, the Twitter empirical data only include one soccer game topic. Although all of the users in this Twitter empirical data follow this soccer game topic, they will easily influenced by other topics, it means that the memory of the soccer game message attenuated quickly, in other words, the memory attenuated parameter should be rather small ($\varepsilon = 0.16$ in this case). On the other hand, in this case, this soccer game started on December 22, 2015, the users who follow this soccer game are interested about this topic, thus, the user takes part in and reposts the soccer game frequently, and it means that the reading probability parameter $\lambda$ is rather large ($\lambda = 1$ in this case). Note that to simplify the physic condition, in this case, we set the whole message number in 1 h as one point in time dimension.

In addition, we compare the memory-attenuated multi-messages propagation probability model with the classical propagation model (i.e. SIR model and threshold model). As shown in Fig. 10, we set optimal parameter in both SIR model and threshold model, the predict condition of threshold model is far away from the Twitter empirical data in the considerable propagation time interval. And the predict condition of SIR model is fit with the Twitter empirical data at the beginning of the spreading, but with the increase of new messages and time passed, the predict condition of SIR model is gradually away from the Twitter empirical data because SIR model ignores the interaction among messages, it is different from the physical condition. Thus, SIR model is unusable when the new messages are large. In this paper, to accord with physical condition, we consider the interaction of each message

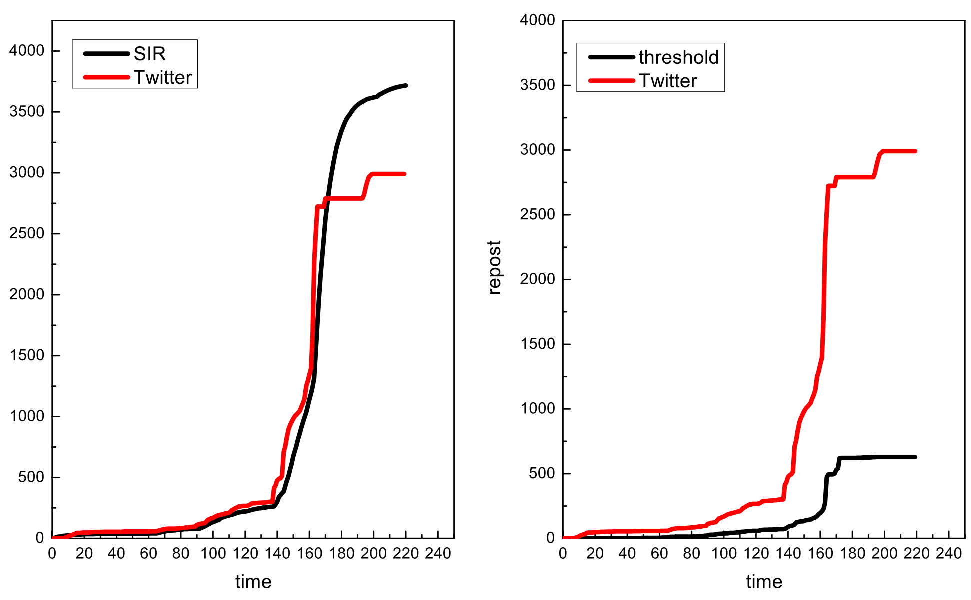


Fig. 10. (Color online) Comparison between the propagation by classic model and actual situation.

Table 1. The relative error between model and the actual data.

| Model | Relative error | The optimal parameters |
|---|---|---|
| Our model | 0.18 | $\varepsilon = 0.16$, $\lambda = 1$ |
| SIR model | 0.4 | The probability of infection is 0.08, the probability of recovery is 0.05 |
| Threshold model | 0.8 | Threshold $T = 2$ |

in our model, note that the correlated value can be obtained by simple learning strategy. As shown in Fig. 8, with the optimal parameter of this model, the predict condition of this model is fit with the Twitter empirical data in the whole considerable propagation time interval. Obviously, the relative error of model is much smaller than the relative error of classical propagation model (i.e. SIR model and threshold model), which shows the validity of model. Table 1 shows the relative error under the optimal parameters of the three models.

## 4. Conclusions

In order to study the spread of topics in online social network, we first propose the SMN framework that takes into account the interaction among multi-messages under the same topic and its influence on transmission. Based on the SMN framework, this paper proposes a multi-message topic dissemination probabilistic model with memory attenuation, introduces the correlation among messages, and different correlation among multi-messages leads to different interactions, and these effects, in turn, affect the spread of information. Through numerical simulation, we found that as the memory parameter $\varepsilon$ of the memory function increases, the user is more interested in the topic, and the message forwarded by the friend is more easily noted by the user and participates in the discussion of the topic, making the spread of the topic wider, and the spread speed faster. When the parameter $\varepsilon$ is large and the reading probability is small, the time of recovery ($R$_time) affects the scope of the topic. The longer the time of recovery ($R$_time), the more the user can spread the topic. However, when the parameter $\varepsilon$ is small, the user is not interested in the topic. The friend's redirection of the topic cannot cause the user's resonance and the influence on the user is small. Therefore, the time of recovery ($R$_time) has little influence on the scope of the communication. When the probability of reading is high, the topic can be read quickly by the user and be concerned by friends during the time of recovery ($R$_time), so in this case, the influence of the time of recovery ($R$_time) on the range of communication is also inconspicuous.

Furthermore,by means of the multi-message topic dissemination probabilistic model with memory attenuation, a curve of message propagation is predicted, which agrees well with Twitter empirical data about a soccer game. The classical SIR epidemic model and threshold model only consider about single message propagation and ignore the correlation of each messages. The model consider about multi-message propagation and the correlation of each messages in the process of propagation. Thus, the model using a weighted message sub-network deals with the propagation of several message, and the weight of message represents the correlation of each message. According to the comparison of the predict message propagation solution with Twitter empirical data, a multi-message topic dissemination probabilistic model with memory attenuation is much more fitted, especially in the multi-message propagation condition.


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