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Network topology and correlation features affiliated with European airline companies

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A R T I C L E I N F O

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The physics information of four specific airline flight networks in European Continent, namely the Austrian airline, the British airline, the France–Holland airline and the Lufthhansa airline, was quantitatively analyzed by the concepts of a complex network. It displays some features of small-world networks, namely a large clustering coefficient and small average shortest-path length for these specific airline networks. The degree distributions for the small degree branch reveal power law behavior with an exponent value of 2–3 for the Austrian and the British flight networks, and that of 1–2 for the France–Holland and the Lufthhansa airline flight networks. So the studied four airlines are sorted into two classes according to the topology structure. Similarly, the flight weight distributions show two kinds of different decay behavior with the flight weight: one for the Austrian and the British airlines and another for the France–Holland airline and the Lufthhansa airlines. In addition, the degree–degree correlation analysis shows that the network has disassortative behavior for all the value of degree k, and this phenomenon is different from the international airline network and US airline network. Analysis of the clustering coefficient ($C(k)$) versus k, indicates that the flight networks of the Austrian Airline and the British Airline reveal a hierarchical organization for all airports, however, the France–Holland Airline and the Lufthhansa Airline show a hierarchical organization mostly for larger airports. The correlation of node strength ($S(k)$) and degree is also analyzed, and a power-law fit $S(k) \sim k^{1.1}$ can roughly fit all data of these four airline companies. Furthermore, we mention seasonal changes and holidays may cause the flight network to form a different topology. An example of the Austrian Airline during Christmas was studied and analyzed.

1. Introduction

Network behaviors emerge across many interdisciplinary sciences, and attract the interest of many researchers in different research fields. A network is usually a set of items, which we will call vertices or sometimes nodes, with connections between them, called edges. Systems in the form of networks are distributed over the world. Examples [1,2] include the Internet, the World Wide Web (WWW), social networks of friends, networks of business relations between companies, neutral networks, metabolic networks, food webs, distribution networks such as blood vessels or postal delivery routes, networks of citations between papers, networks of paper collaborators, networks of publication download frequency [3,4], and many others. Even in the microscopic scale, such as in nuclear fragmentation produced in heavy ion collisions, the hierarchial power-law distribution of nuclear fragments emerges around the nuclear liquid gas phase transition, which also shows a similar character to the scale free network [5,6].

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Table 1
Some variables of the Austrian Airline in European Continent: (1) numbers of the airports $N_{airport}$; (2) the numbers of fights M_{flight}; (3) the flight density per airport ρ; (4) the average incoming, outgoing and undirected flight degree $\langle k_{in}\rangle$, $\langle k_{out}\rangle$ and $\langle k_{all}\rangle$; (5) γ_{in}, γ_{out} and γ_{all}, which represents the exponent of first segment of incoming, outgoing and undirected flight degree distribution, respectively; (6) the exponents of outgoing weighted flight distribution $\gamma_{flight\text{-}out}$; (7) the average shortest distance L_s; (8) the clustering coefficient C of the system; (9) the assortative coefficient r

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
$N_{airport}$	130	118	120	121	126	124	129
M_{flight}	721	698	714	709	722	624	647
ρ	5.538	5.915	5.950	5.860	5.73	5.032	5.016
$\langle k_{in}\rangle$	2.359	2.432	2.474	2.445	2.408	2.341	2.397
$\langle k_{out}\rangle$	2.378	2.453	2.443	2.425	2.389	2.322	2.435
$\langle k_{all}\rangle$	2.446	2.474	2.467	2.446	2.428	2.387	2.450
γ_{in}	2.80	2.60	2.46	2.61	2.65	2.65	2.77
γ_{out}	2.86	2.56	2.60	2.62	2.85	2.87	2.83
γ_{all}	2.428	2.443	2.458	2.464	2.498	2.605	2.507
$\gamma_{flight\text{-}out}$	0.678	0.616	0.715	0.886	0.955	1.189	0.874
L_s	2.31	2.29	2.26	2.28	2.30	2.25	2.28
C	0.122	0.138	0.135	0.122	0.142	0.150	0.138
r	−0.583	−0.571	−0.566	−0.577	−0.582	−0.619	−0.584

Airline networks, as a part of the urban traffic system, are a typical open complex huge system. It is a complex system, which is composed of routes in certain regions connected according to given laws, including airports, airlines, aircraft and other factors. If we treat airports as nodes, flights between the airports as edges, the throughput of the airports as vertex weight, and the number of passengers (or flight distance) as link weight, the airline network could be viewed as a weighted complex network. It has features such as the statistical character of networks' activities, the sparsity of networks' connections, the complexity of the connecting structure, the time and space complexity of networks, and the synchronous movement of network nodes (or edges). As a complex huge system, an airline network has all sorts of problems. If we only consider one aspect, for instance airport facilities or networks, it is impossible to solve these problems perfectly. We should develop research in-depth to find out the formation mechanisms and evolution laws of airline network microscopically and macroscopically, and combine the multidisciplinary advantages of cross-mathematical science, systems science, management science, information science and traffic engineering, band the traffic flow model and the characteristics of complex networks together. By these complex studies, people can try to reveal the general evolution laws of future airline networks and fully grasp the airline network's relative stability in space–time and the complexity of the structure, and develop a profound understanding of various factors of the air transport system constraints. Finally, a mathematical and physical description of the general airline network evolution laws could be reached, and it will provide support of design optimization and decision-making in a future airline network.

From the beginning of the 21st century, people made an attempt to find a new perspective to study the structural characteristics of airline networks. Guimera and Amaral's research on the International Airline Network of 2002 [7] showed two important conclusions: (1) the world airline network was a small-world network which followed a power-law degree distribution and declines in the number of betweenness; (2) the city which had the largest node degree was not always one of the hub cities (the largest betweenness). Cai et al. studied the US and China flight networks in succession, and had achieved similar conclusions [8]. The flights on one day or one week are viewed as link weight and its distribution has demonstrated a power-law distribution. The link weight is proportional to the degree of airports (i.e. the number of skyways). Henceforth, Bagler [9] had applied an analogous statistical method to study the Indian national airport network. A conclusion similar to the China airport network had been achieved. However, our present work is different in motivation and results. The previous flight network analysis involved an entire national or international airport network, which was not concerned with detailed information of the fights, which were operated by a specific airline company. These national- or world-wide flight networks are large-scale [8,10–12] but they are the result of a collective role by the various airline company networks. Therefore, it is of interest to survey a particular airline flight network, instead of an entire national or international-wide flight network, and check if there is a similar or different network behavior between a large scale multiple-airline network and a sole airline network. Furthermore, the main function of a specific airline company is more important in the structure of an airline network. Based upon the above motivations, we will investigate some running airline networks which are composed of the flights affiliated with some specific airline companies in the present work. As examples, we have investigated four flight networks of European airline companies, namely the Austrian Airline (OS), the British Airline (BA), the France–Holland Airline (AF-KLM) and the Lufthhansa Airline (LH). The flight information is available in the web page, http://www.aua.com/, http://www.ba.com/, http://www.af-klm.com/and http://www.lufthansa.com/.

In the four flight networks, the airports can be represented by the vertices, and the flights connecting two airports by edges. Therefore, some features of the structure of flight networks have been recognized: (1) the network is directional. All the flights are directed, sorted as outgoing and incoming. (2) the network has weight. To reflect how busy a certain line is, it is important to record the exact number of flights between any given airport i and j [8], even to record the seat numbers available in different flights [10]. (3) the network may be a little different day by day in a whole week. Hence, the weekly flight information partially involves information on evolution of the flight network. Our data contain a whole week's information. The detailed numbers of the airports and flights are listed in Tables 1–4 for OS, BA, AF-KLM and LH, respectively. From the

Table 2
Same as Table 1 but for the British Airline

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
$N_{airport}$	320	318	320	319	327	315	314
M_{flight}	2161	2151	2145	2161	2152	1782	1881
ρ	6.75	6.76	6.70	6.77	6.58	5.66	5.99
$\langle k_{in} \rangle$	2.574	2.565	2.503	2.620	2.533	2.459	2.495
$\langle k_{out} \rangle$	2.605	2.626	2.634	2.607	2.590	2.556	2.591
$\langle k_{all} \rangle$	2.747	2.667	2.624	2.664	2.635	2.590	2.617
γ_{in}	2.75	2.77	2.91	2.94	2.73	2.84	2.83
γ_{out}	2.78	2.99	2.80	2.94	2.98	2.61	2.31
γ_{all}	2.791	2.790	2.733	3.073	2.894	2.471	2.864
$\gamma_{flight\text{-}out}$	0.736	0.693	0.621	0.717	0.749	0.792	0.807
L_s	2.64	2.65	2.66	2.64	2.66	2.62	2.64
C	0.124	0.121	0.131	0.139	0.116	0.103	0.118
r	−0.625	−0.615	−0.637	−0.604	−0.616	−0.672	−0.656

Table 3
Same as Table 1 but for the France–Holland Airline

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
$N_{airport}$	348	346	341	353	347	340	335
M_{flight}	3853	3783	3794	3835	3851	3052	3202
ρ	11.07	10.93	11.13	10.86	11.10	8.97	9.59
$\langle k_{in} \rangle$	4.519	4.482	4.489	4.469	4.491	4.519	4.448
$\langle k_{out} \rangle$	4.743	4.605	4.737	4.673	4.664	4.591	4.639
$\langle k_{all} \rangle$	4.644	4.543	4.644	4.561	4.592	4.559	4.608
γ_{in}	1.83	1.84	1.82	1.85	1.83	1.65	1.71
γ_{out}	1.74	1.76	1.73	1.76	1.75	1.72	1.72
γ_{all}	1.833	1.844	1.823	1.856	1.837	1.665	1.892
$\gamma_{flight\text{-}out}$	0.985	1.212	1.065	1.169	1.115	1.244	1.168
L_s	2.19	2.21	2.20	2.20	2.20	2.22	2.20
C	0.371	0.374	0.374	0.376	0.377	0.387	0.382
r	−0.494	−0.487	−0.504	−0.496	−0.502	−0.515	−0.506

Table 4
Same as Table 1 but for the Lufthhansa Airline

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
$N_{airport}$	430	425	429	424	420	420	426
M_{flight}	4643	4606	4621	4602	4643	3769	4054
ρ	10.80	10.84	10.77	10.85	11.05	8.97	9.52
$\langle k_{in} \rangle$	4.398	4.464	4.424	4.460	4.472	4.336	4.471
$\langle k_{out} \rangle$	4.398	4.411	4.446	4.332	4.538	4.272	4.548
$\langle k_{all} \rangle$	4.526	4.546	4.527	4.528	4.638	4.462	4.596
γ_{in}	1.84	1.89	1.73	1.84	1.99	1.98	1.80
γ_{out}	1.81	1.81	1.90	1.98	1.98	1.86	1.84
γ_{all}	1.755	1.704	1.853	1.938	1.865	1.958	1.936
$\gamma_{flight\text{-}out}$	1.023	1.033	1.049	1.043	0.939	1.039	1.069
L_s	2.900	2.910	2.910	2.920	2.920	2.890	2.880
C	0.376	0.382	0.362	0.384	0.376	0.368	0.368
r	−0.321	−0.321	−0.324	−0.320	−0.321	−0.318	−0.333

listed flight and airport numbers for above four companies, OS is the smallest, BA is the second, AF-KLM is the third, and LH is the largest. Furthermore, we can define the flight density (ρ) as the ratio of flight numbers to the airport number, which is also listed in the tables. Essentially, we can sort four companies into two classes, according to ρ, one is for OS and BA whose ρ is around 5–7 and another for AF-KLM and LH whose ρ is around 9–11. As we can see in next sections, the topological behavior can be also classified into two classes, one for OS and BA, another for AF-KLM and LH as the flight density does. From Monday to Sunday, the flight number and airport number show changes. Usually, there are fewer flights and airports available in weekend days, especially these are the least on Saturdays. Flights and airports do not change much on different working days, though it somehow displays the largest on Mondays and Fridays. The above changes of flight and airport numbers are related to the human activity of leisure and business. Monday corresponds to the busiest flight transportation day, since people prefer to travel and work, while Saturday corresponds to the most unoccupied flight transportation day, since people prefer to rest.

The paper is organized as follows. First we present the topology of each airline company, and compare the differences between a larger company and a smaller company in Section 2. We give the results of the flight weight distributions in

Section 3. Section 4 analyzes the correlation features and dynamics behavior about the four flight companies at a distance. Finally, as an example of the Austrian Airline company, we investigate the dramatic changes about topology, weight, correlation dynamics behavior during the Christmas holiday in Section 5, and finally a summary is given in Section 6.

2. Topology of the four flight networks

The vertex degree distribution function P_k gives the probability that a randomly selected vertex has exactly k edges [14]. Two different kinds of degree distributions of the four airline companies, namely $P_k(in)$ and $P_k(out)$ are shown in Figs. 1 and 2. $P_k(in)$ and $P_k(out)$ represent the frequencies of incoming and outgoing of flights, respectively. Note that the present degree distribution is not cumulative distribution. Even though the statistical fluctuation could be large in degree distribution, in comparison with the cumulative distribution, distribution can give a direct probability how many flights of the four airline companies are coming or taking off. A power-law branch is seen in smaller k region of Figs. 1 and 2, but there exists cut-off when k becomes larger. This is distinct for OS and BA, namely the flat tail degree distribution when $k \geq 5$ for OS and $k \geq 7$ for BA, which is basically related to some of the largest airports which serve for OS and BA. With an increase of flight density, AF-KLM shows a discontinuity around $k \geq 8$, and while LH becomes more or less continuous in a large k region and tends to be flat when $k \geq 15$. Different cut-offs in different airline companies are basically related to the distribution density of flights. That is to say, the distribution density of the Austrian Airline network is similar to that of the British Airline network, which has a similar incoming degree k_{in}, and the distribution density of the Lufthhansa Airline network is similar to that of the France–Holland Airline network. In other words, the larger the airline companies, the larger the cut-off value. From this viewpoint, we can say the phenomenon of cut-off is related to the finite size effect. To check if the power law distribution could describe the degree distributions well, we used the Kolmogorov-Smirnov test for the degree distributions among four airline companies and among different weekdays. From the degree distributions, we did observe that the distribution is not a normal distribution and favors a power law distribution or log-normal distribution. However, as the paper of Clauset et al. shows [13], sometimes it is not so easy to determine whether the distribution is power law or log-normal. But, referring to previous publications on international or national-wide flight networks [8–12], we would say a power law is more reasonable. In the following, we can extract the exponents of the degree distribution γ_{in}, γ_{out} and γ_{all} for a small degree region which the four airline flights cover. Exponents in each day of the week are listed in Tables 1–4. From the figures, γ_{in} is 2.80 and 2.75 for the OS and BA, respectively; it is 1.83 and 1.84 for the AF-KLM and LH, respectively. While, γ_{out} is 2.86 and 2.75 for the OS and BA, respectively; it is 1.74 and 1.81 for the AF-KLM and LH, respectively. From the above exponents, we could say the OS and BA is one topological class, and the AF-KLM and LH is another topological class. In addition, we also checked from the Kolmogorov-Smirnov test, and found that the power-law exponents can be classified into two classes for OS/BA and AF-KLM/LH, respectively. When not considering the direction, the average degree of each airline company in one week is $\langle k\rangle_{\text{all}}(\text{OS}) = 2.443$, $\langle k\rangle_{\text{all}}(\text{BA}) = 2.649$, $\langle k\rangle_{\text{all}}(\text{AF-KLM}) = 4.593$ and $\langle k\rangle_{\text{all}}(\text{LH}) = 4.546$, respectively, for four airline companies. That means one airport is, on average, linked to two or three other airports for the flights affiliated with the Austrian Airlines or the British Airlines, while it is linked to more than four other airports for the flights affiliated with the Lufthhansa Airlines and the France–Holland Airlines. Similarly, $\langle k_{\text{in}}\rangle$ and $\langle k_{\text{out}}\rangle$ on each day have been achieved, respectively, as listed in Tables 1–4. From the average degree, OS/BA is one class and AF-KLM/LH is another, as sorted by the power-law exponents.

In a random graph of the type studied by Erdös and Rényi, each edge is present or absent with equal probability, and hence the degree distribution is binomial or Poisson distribution in the limit of large graph size. Real-world networks are mostly found to be very unlike the random graph in their degree distributions. The degrees of the vertices in most networks are highly right-skewed. This is the case of the present four flight networks. From the exponents γ of different days in a week as shown in Tables 1–4, we can find that exponents $\gamma_{\text{in,out}}$ and γ_{all} are different day by day with fluctuations in a certain area.

In many social networks, there exists a clique form which can be represented by circles of friends or acquaintances. Quantitatively, this inherent tendency to cluster can be expressed by the clustering coefficient [15]. In an airline network, a clustering coefficient represents the width of transportation which presents the average clustering degree of network constructed by the nodes of navigating airports and neighboring airports. That is, the index denoting network maturity. For a selected vertex i of the network, it has k_i edges, which we call the nearest neighbors of i. In this case, the maximal possible edges among k_i neighbors are $k_i(k_i - 1)/2$. If we use N_{real} to denote the number of edges that actually exist, the clustering coefficient of vertex i can be written as

$$C_i = \frac{N_{\text{real}}}{k_i(k_i - 1)/2} \tag{1}$$

and the clustering coefficient of the entire network is defined as $C = \frac{1}{N}\sum_i C_i$. The values of C for each day are listed in the tables. The average clustering coefficient C of the flight network in a week is $C = 0.135$, 0.122, 0.378 and 0.374, for OS, BA, AF-KLM and LH airlines, respectively. To look for differences in a random network, we compare C of our flight networks with that of a random graph which has the same average N and $\langle k\rangle$. The results C_{rand} of random graphs are 0.029, 0.012, 0.011 and 0.008 for OS, BA, AF-KLM and LH airlines, respectively. Obviously, C in our flight networks are much larger than those in random graphs.

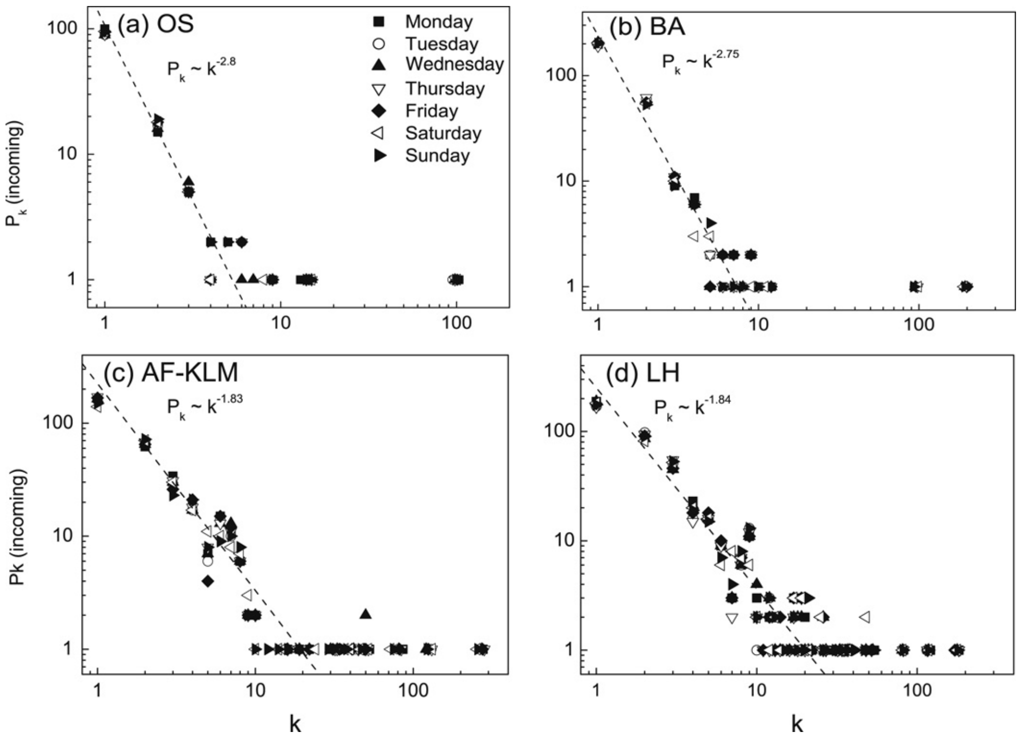

Fig. 1. Incoming-degree distribution for each day during a week. (a) In the case of the Austrian Airline, the average exponent of incoming degree distribution $\gamma_{in} = 2.80$; (b) in the case of the British Airline, the average exponent of incoming degree distribution $\gamma_{in} = 2.75$; (c) in the case of the France–Holland Airline, the average exponent of incoming degree distribution $\gamma_{in} = 1.83$; (d) in the case of the Lufthhansa Airline, the average exponent of incoming degree distribution $\gamma_{in} = 1.84$.

The average shortest-path length between any two airports in the system can be characterized by so-called "diameter" in small-world networks [16], which is defined as

$$L_s = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij}, \tag{2}$$

where d_{ij} is the minimum number of edges traversed from vertex i to vertex j. The diameter of the flight network reflects the average number of least possible connections between any two airports. The average shortest-path distance in an airline network represents the depth of transportation. While the depth is smaller, the transportation is more rapid and easier. Thus the connection between two nodes should be as small as possible. The average shortest-path length L_s of the Austrian Airline, the British Airline, the France–Holland Airline and the Lufthhansa Airline is 2.28, 2.64, 2.20 and 2.90, respectively. This implies that there will be basically no more than two connections from one airport to another airport, by taking the Austrian flights or the France–Holland flights, while not more than three connections by taking the Lufthhansa flights or the British flights. Using the same approach, we compare the L_s of our flight network with that of random graphs. The values of random graphs are 5.19, 5.90, 3.98 and 4.29, respectively. Hence, the diameter of our flight network is significantly smaller than the one of the random graph with the same nodes and mean degree.

Since the four flight networks affiliated with four specific companies have both significantly smaller average shortest distance and larger clustering coefficient, these networks are considered as a small-world network. Furthermore, comparing the average degree $\langle k \rangle$ and exponent of distribution γ, we find that the Austrian Airline and the British Airline have a similar topology, while the France–Holland Airline and the Lufthhansa Airline have another, similar, topology. From the viewpoint of airport distribution and density, we can classify the four flight networks into two groups, one for OS and BA, another for AF-KLM and LH. Actually, even though OS has relative fewer airports and flights in comparison with BA, the flight density per airport is similar, i.e. from 5 to 7; Similarly, there is close flight density per airport for AF-KLM and LH, i.e. from 9 to 11. In this viewpoint, the flight density per airport could be an important ingredient to classify the network topology.

3. The weight features of four airline companies

Since the flight network is involved in transportation flux, the weight is important and can reflect some information of the whole network. As shown in Fig. 3, the flight weight distribution in a week shows a linear behavior in a double logarithmic plot for the small weight branch. But we should be careful that this linearity only fits a few points, especially for the OS and

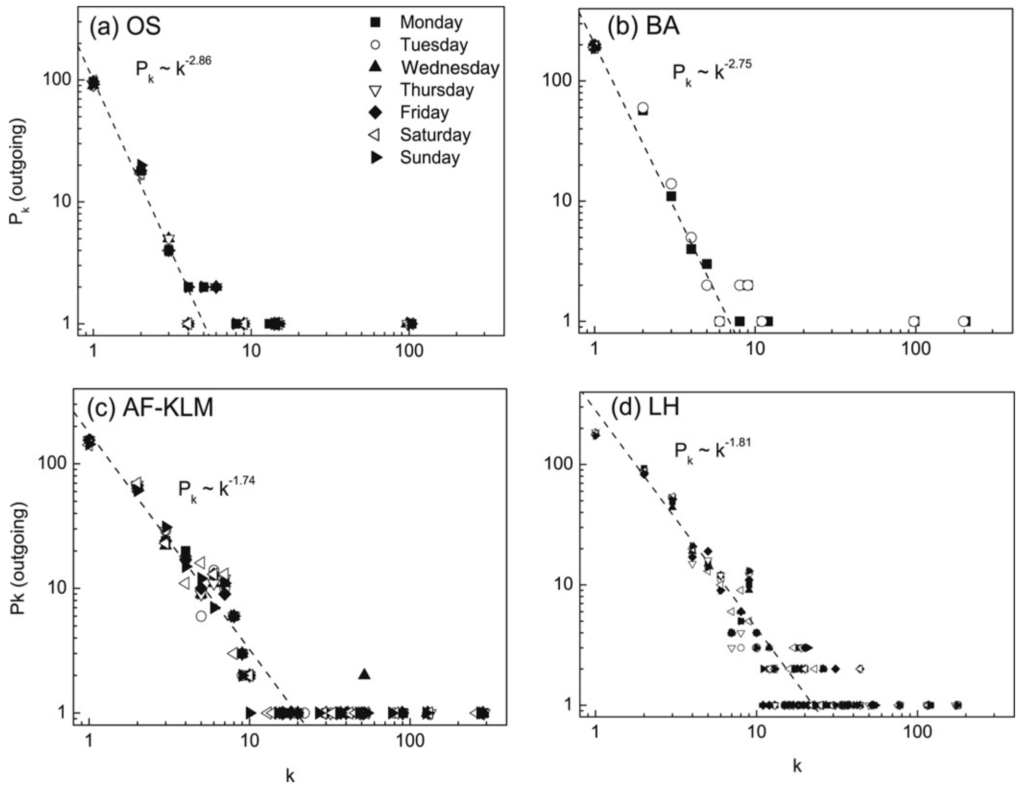

Fig. 2. Outgoing-degree distribution for each day during a week. (a) In the case of the Austrian Airline, the average exponent of outgoing degree distribution $\gamma_{out} = 2.86$; (b) in the case of the British Airline, the average exponent of outgoing degree distribution $\gamma_{out} = 2.75$; (c) in the case of the France–Holland Airline, the average exponent of outgoing degree distribution $\gamma_{out} = 1.74$; (d) in the case of the Lufthhansa Airline, the average exponent of outgoing degree distribution $\gamma_{out} = 1.81$.

BA. However, to quantitatively compare the possible difference for the weight distributions among four flight networks, we assume a power law distribution which is only adopted in the small weight branch, namely

$$P_s \sim s^{-\gamma_{\text{flight}}}, \tag{3}$$

to see the exponent γ_{flight}, where s is the exact number of flights between any given airport i and j. The outgoing network exponents, $\gamma_{\text{flight-out}}$, of different days in a week are shown in Tables 1–4. The mean exponents of four airline companies are 0.85, 0.73, 1.13 and 1.03, respectively, for OS, BA, AF-KLM and LH. Essentially, we could say the former two are one class with smaller exponent, and the latter two are another, with a larger exponent. These two classes for the exponents of weight distribution are also consistent with two classes for the exponents of the degree distributions, which was shown in Fig. 2.

4. Correlation and dynamics behavior of four flight networks

Many networks show assortative mixing on their degrees, i.e., a preference for high-degree vertices to attach to other high-degree vertices, while others show disassortative mixing, which corresponds to high-degree vertices attaching to low-degree ones. Quantitatively, the degree–degree correlation coefficient (also called assortative coefficient) can be written as

$$r = \frac{\frac{1}{M}\sum_i j_i k_i - [\frac{1}{M}\sum_i \frac{1}{2}(j_i + k_i)]^2}{\frac{1}{M}\sum_i \frac{1}{2}(j_i^2 + k_i^2) - [\frac{1}{M}\sum_i \frac{1}{2}(j_j + k_i)]^2}, \tag{4}$$

where j_i and k_i are the degrees of the vertices at the ends of the ith edge, with $i = 1, \ldots, M$. The value of M is total edges of network. As Newmann showed, the values of r of the social networks have significant assortative mixing. By contrast, the technological and biological networks are all disassortative [17]. In this work, we also check the coefficient r, and we list those values of each day in Tables 1–4. As we expected, the values are all negative, which means the four flight networks are disassortative. In other words, the larger airports are likely to link to smaller airports. This fact is decided by the scale of the network and their own characteristics.

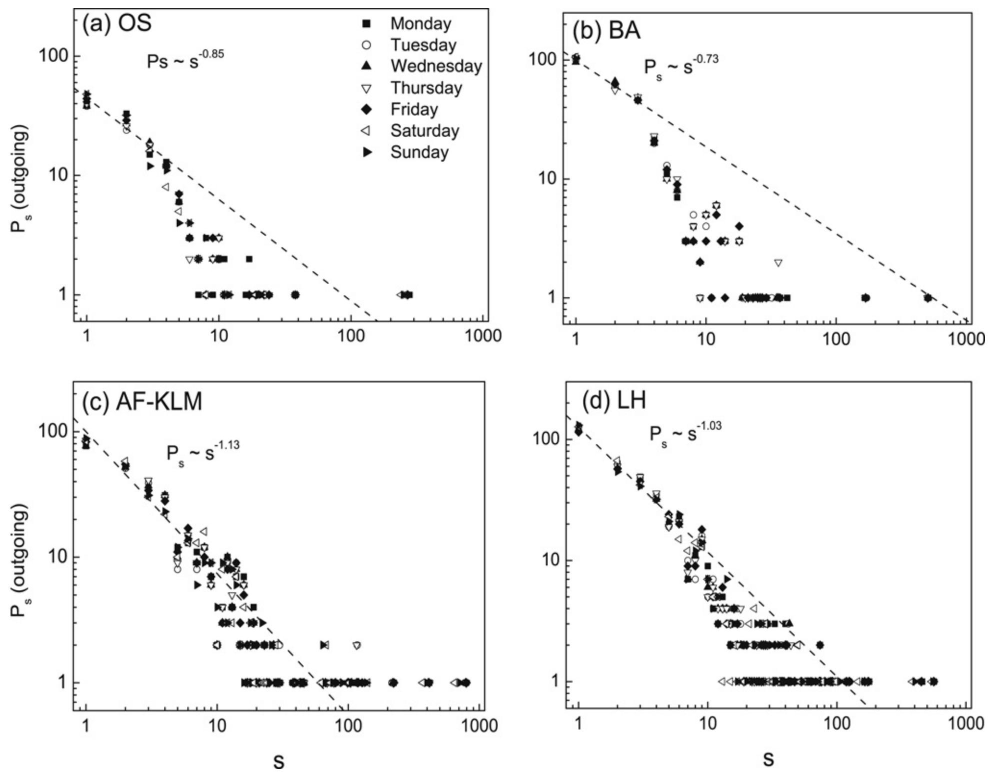

Fig. 3. Outgoing flight distribution of four airline companies for weighted flight for each day during a week. (a) In the case of the Austrian Airline, the average exponent of outgoing weighted flight distribution $\gamma_{\text{flight-out}} = 0.85$; (b) in the case of the British Airline, the average exponent of outgoing weighted flight distribution $\gamma_{\text{flight-out}} = 0.73$; (c) in the case of the France–Holland Airline, the average exponent of outgoing weighted flight distribution $\gamma_{\text{flight-out}} = 1.13$; (d) in the case of the Lufthhansa Airline, the average exponent of outgoing weighted flight distribution $\gamma_{\text{flight-out}} = 1.03$.

Besides calculating the above assortative coefficients r to represent degree–degree correlation, some references demonstrated that the connecting probability between node degree k and node degree k' is related to k [18,19]. Quantitatively, this relation can be denoted by condition probability, ie. $k_{nn}(k) = \sum_{k'} k'P\langle k'|k\rangle$. Statistically analyzing the real network, we can assay the average nearest neighbor's degree of node i to get the correlation. Quantitatively, average degree of whole neighbor nodes of i can be written as

$$k_{nn,i} = \frac{1}{k_i} \sum_{j \in N_i} k_j. \tag{5}$$

Further, the average neighbor degree of whole nodes, whose degree is k, can be written as

$$k_{nn}(k) = \frac{1}{N_k} \sum k_{nn,i}, \tag{6}$$

where N_k is the number of nodes with the degree of k.

Fig. 4 shows the relation between whole nodes degree and their nearest neighboring degree. From this figure a degree–degree correlation can be presented directly. If $k_{nn}(k)$ rises, followed by k, means that high-degree nodes prefer to connect with other high-degree nodes, i.e. network reflects assortativity. Whereas, if $k_{nn}(k)$ decreases, followed by k, it means that high-degree nodes prefer to connect with low-degree nodes, i.e. the network shows disassortativity.

Comparing the degree–degree correlation of an International airline and a US airline, we have found that the disassortativity presented by four airline networks in European Continent is different from others. As to an International airline, it is assortative when the degree is smaller; but it shows no clear correlation when the degree becomes larger [7]. As to a US airline, $k_{nn}(k)$ increased by k when $k < 30$; but $k_{nn}(k)$ tends to decrease when $k \geq 30$ [20]. The difference between our flight networks and an International or US airline network may come from two reasons. One is that the size of network is different, and the other, is that the network structure is different. The first two airline networks are based on certain countries, but in our present work, airline networks affiliated with specific companies are studied.

Clustering-degree correlation means the relation between clustering coefficients with k-degree and k. Fig. 5 shows the scattering plots of the clustering coefficient for a uni-directional flight network of each day in a week as a function of the

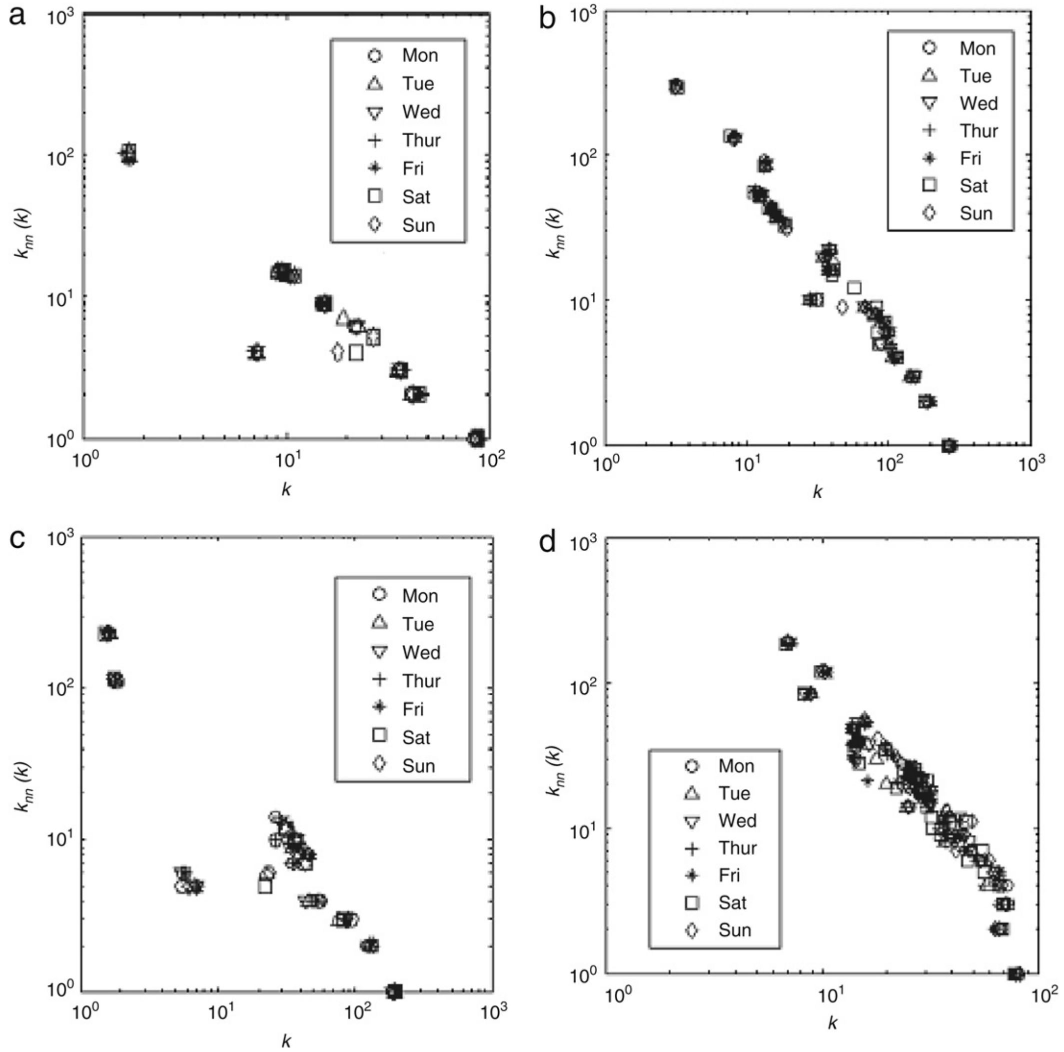

Fig. 4. The average nearest neighbors degree as a function of the vertex degree. (a) In the case of the Austrian Airline, the average assortative coefficient for one week $r = -0.583$; (b) in the case of the France–Holland Airline, the average assortative coefficient for one week $r = -0.501$; (c) in the case of the British Airline, the average assortative coefficient for one week $r = -0.632$; (d) in the case of the Lufthhansa Airline, the average assortative coefficient for one week $r = -0.323$.

vertex degree. There is a negative clustering-degree correlation. Similar to the degree distribution (Figs. 1 and 2), the scatter plots in Fig. 5(a) and (b) keep approximately linear in double logarithmic plots for OS and BA, where we can presume that there is again, a power-law correspondence between $C(k)$ and k. However in Fig. 5(c) and (d) the scatter plots are more flat in the area of small k with a larger dispersion, but there is a power-law decay with the exponent $\sim$1.3 for $k \geq 20$. The small-k branch corresponds to the majority of airports with a few links to other airports, each such airport i has a clustering coefficient close to 1. The high-k airports include many large airports, and thus, their neighbors are not necessarily linked to each other, resulting in a smaller C_k. In Fig. 5, a power-law decay of high-k branch indicates that a hierarchical organization [21] for larger airports, in contrast to the k-independent C_k which was predicted by the scale-free networks as in small-k branch. This kind of hierarchical organization, which presents nodes for a small degree has a high clustering coefficient, and belongs to small, highly-connected modules. It is an obvious case in the France–Holland airline and the Lufthhansa airline. By contrast, the central hub nodes are used to connect different modules which have a larger degree, while the clustering coefficient is low. Note that ER random graphs and the BA scale-free network do not have a hierarchical topology.

In weighted network, node strength or node weight S_i is denoted as

$$S_i = \sum_{j \in N_i} w_{ij}, \qquad (7)$$

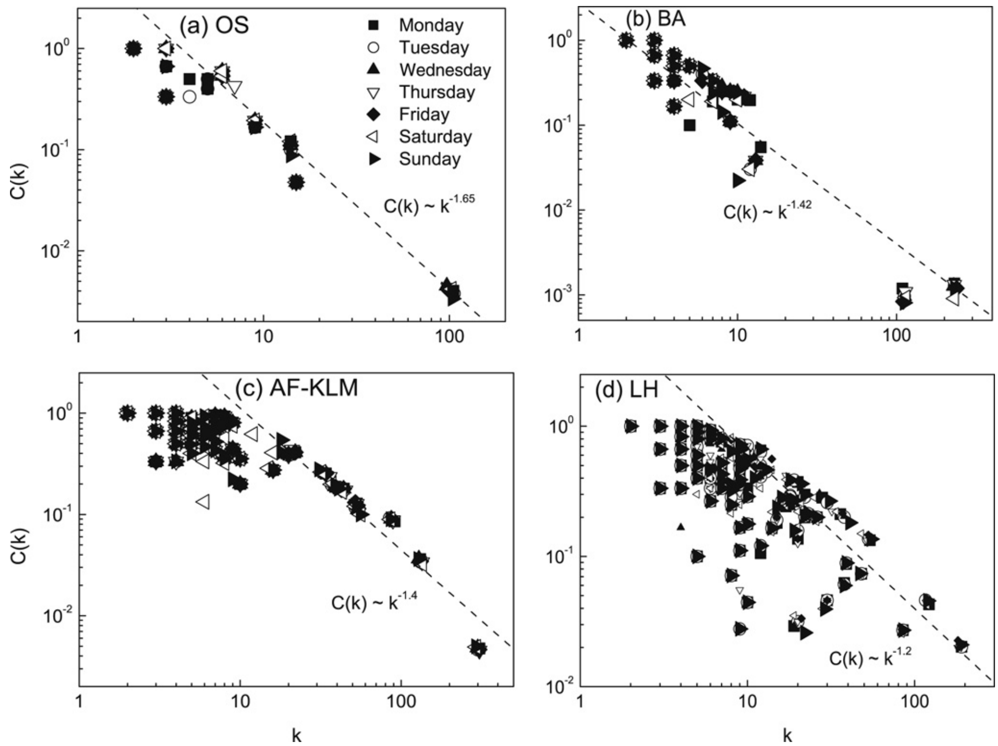

Fig. 5. The clustering coefficient as a function of the vertex degree. The dotted line represents the tends of $C(k)$ for large k. The slope is given in the figure. The scatter plots in (a), (b) are approximately linear for OS and BA. However in (c), (d) the scatter plots are more flat in the area of small k with a larger dispersion, but there is a power-law decay with the exponent $\sim$1.3 for $k \geq 20$.

where N_i is the nearest neighbors set of node i. The node strength takes the number of nearest neighbors as well as the weight between node i and nearest neighbor into account. When the node strength is a linear function of the degree, i.e.

$$S(k) \approx \langle w \rangle k, \tag{8}$$

where $\langle w \rangle$ is the average of strength, we say the strength is independent of network topology. On the contrary, the strength follows a power law where its exponent $\beta \neq 1$, namely

$$S(k) \approx Ak^{\beta}, \tag{9}$$

we say the strength is dependent of network topology. Similar to the US airline network, the average weight is a nonlinear correlation to vertex-degree k in our present four flight networks (see Fig. 6). The node strength has approximately power law behavior, i.e. $s(k) \sim k^{\beta}$, where β of all four flight networks are close to 1.1 rather than 1. This exponent, which is larger than 1, reflects that the larger the airport, the stronger the capability of managing transportation flux.

5. Topology of the Austrian Airline in Christmas holiday

The arranged flight schedule and transportation loads can be changed by any airline company because of seasonal changes and holidays. Any airline company wants to reduce transportation costs on the premise of not influencing travelers, so that they can use airline transportation efficiently. So we need to study ulteriorly topology of the airline company in the case of holidays. As an example, we select the Austrian Airline to record real-time data during the Christmas holiday of a week in 2007 (19th Dec.–25th Dec.) to investigate the network's behavior.

Table 5 recorded flights number, airports number, mean degree, exponents of power-law of degree distribution, clustering coefficients, average shortest distance, exponents of weighted distribution of outgoing flight and assortative correlation coefficient during the Christmas holiday. By analyzing this table, we found that airport number and flight number at Christmas are much less than those of work days. The flight density ρ also greatly decreases. Similarly, the average degree of $\langle k_{in} \rangle$, $\langle k_{out} \rangle$ and $\langle k_{all} \rangle$ decreases dramatically on the days of the 23th and 24th. If we look more carefully, $\langle k_{in} \rangle$ shows a minimal on Dec 24th and $\langle k_{out} \rangle$ shows a minimal on Dec 25th. For the exponents of the weighted degree distributions, the decrease is also distinct during Christmas. As to clustering the coefficient C during Christmas, the mean value is less than that of working days. Specifically, this value is the least before or after the 24th. It is explained that the number of hub nodes existing in the airline network dramatically reduces. However, the values of L_s are larger than those of working days, which reflects that the average shortest-path length becomes longer due to many airports closing during the holidays. For

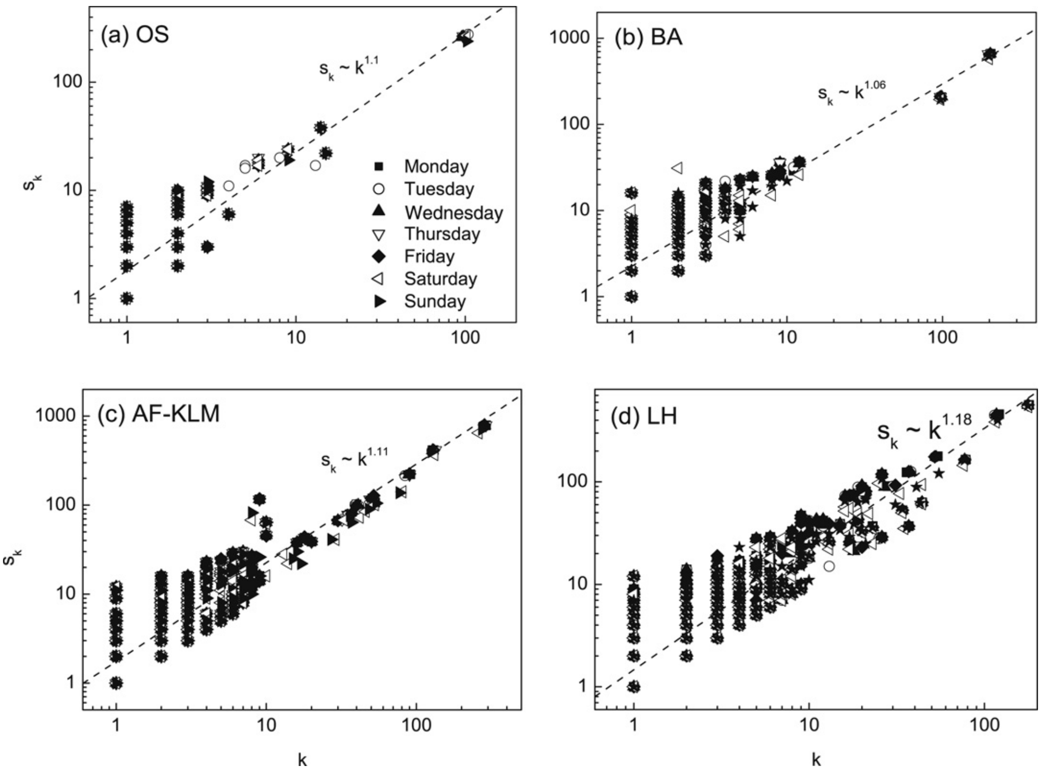

Fig. 6. The correlation of node strength and degree. (a) the Austrian Airline, $s(k) \sim k^{1.10}$; (b) the British Airline, $s(k) \sim k^{1.06}$; (c) the France–Holland Airline, $s(k) \sim k^{1.11}$; (d) the Lufthhansa Airline, $s(k) \sim k^{1.18}$.

Table 5
Same as Table 1 but during the Christmas week

	Wed/19th	Thu/20th	Fri/21st	Sat/22nd	Sun/23rd	Mon/24th	Tue/25th
$N_{airport}$	113	113	112	104	99	84	69
M_{flight}	592	596	591	455	463	295	259
ρ	5.239	5.274	5.277	4.38	4.68	3.512	3.754
$\langle k_{in} \rangle$	2.287	2.333	2.318	2.220	1.740	1.632	2.235
$\langle k_{out} \rangle$	2.266	2.400	2.340	2.155	2.300	2.214	1.650
$\langle k_{all} \rangle$	2.301	2.354	2.339	2.192	2.141	2.095	2.145
γ_{in}	2.46	2.61	2.65	2.65	2.77	2.8	2.6
γ_{out}	2.6	2.62	2.85	2.87	2.83	2.86	2.56
γ_{all}	2.458	2.464	2.498	2.605	2.507	2.428	2.443
$\gamma_{flight\text{-}out}$	1.903	1.709	1.626	1.883	1.474	1.504	1.765
L_s	2.228	2.213	2.222	2.179	2.444	2.442	2.456
C	0.083	0.103	0.092	0.067	0.030	0.048	0.010
r	−0.651	−0.643	−0.648	−0.705	−0.880	−0.887	−0.498

degree–degree correlation function, r tends to −1 on Dec 23 and 24, it indicates that the network topology changes from the hub-and-spoke structure to a star structure, that is to say, only a few main airports, such as Vienna International airport can be run with other airports on that special day. From this special comparison, we observe a dramatic change in the flight network's behavior during holidays.

6. Summary

In summary, we investigated four flight networks designed by some specific airline companies, namely the Austrian Airline, the British airline, the France–Holland airline and the Lufthhansa airline. The corresponding daily flight numbers for the above companies is from ∼620 to ∼4600. Their degree distributions of fewer-connected airports show a power-law behavior for incoming, outgoing and undirected flight networks. It demonstrates that there is a similar power-law exponent for the Austrian Airline and the British airline, while there is another, similar, exponent for the France–Holland airline and the Lufthhansa airline. This may relate to the fact that the daily flight density for OS and BA are similar, and the daily flight density for AF-KLM and LH are also similar. In addition, the corresponding weighted flight distributions in the smaller weights branch are a similar class for the OS and the BA, and the class for the AF-KLM and the LH is also similar. The network analysis results demonstrate that these flight networks have small-world properties, such as high clustering coefficient and small diameter. Speaking in more detail, we found that the clustering coefficient C is greatly larger than that of a random network with the same N and $\langle k \rangle$ while the diameter L_s of the flight network is significantly smaller than the value of the same random network. Our specific airline flight networks have small-world and scale-free behavior, similar to the larger scale world-wide or nation-wide airline networks, which reflects that the coherent role of different airline companies does not interfere with basic network characters of the airline network. Of course, some quantitative exponents remain slightly different. Furthermore, the network shows disassortative behavior for all the value of degree k in the four presented airline flights. This disassortativity indicates that large airports are likely to link to smaller airports. The power-law decay behavior of the clustering coefficient against the degree for high-k nodes reflects that the larger airports reveal a hierarchical organization. In addition, the correlation of node strength and degree shows a power-law-like behavior with an exponent around 1.1 instead of 1, which reflects that the larger the airport, the stronger the capability of managing transportation flux. Finally, we discuss the network behavior during the Christmas days for the Austrian Airline which shows distinct differences in the mean degree of incoming and outgoing flights, the clustering coefficient, the diameter as well as the degree–degree correlation function. It reflects the dramatic change of human activities in seasons holidays. From the present work, we note that each airline network shows a rich network structure. It is possible that further systematic studies could provide some help for designing, extending or optimizing some airline networks for some new airline companies.

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